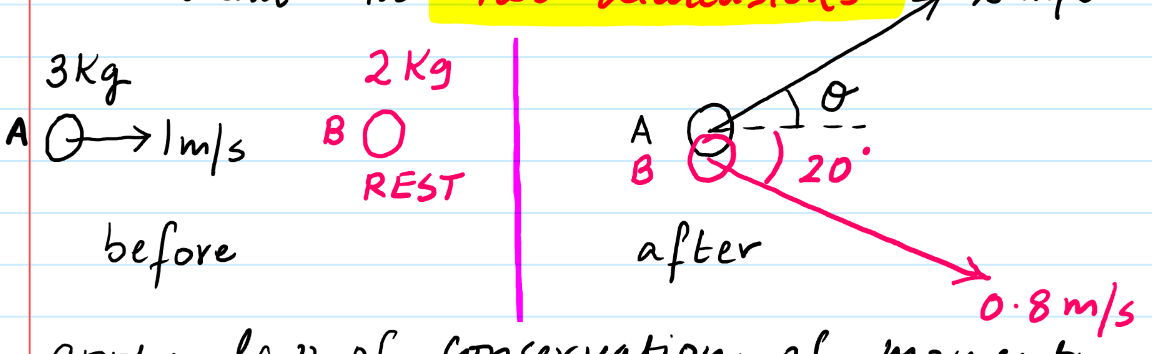
How to apply Law of Conservation of momentum in "Two dimensions"



apply law of conservation of momentum to find x and θ ?

① L.O.C.O.M in the horizontal plane

$\xrightarrow{+}$ $(3)(1) + (2)(0) = (3)(x\cos\theta) + (2)(0.8\cos 20)$

$3 = 3x\cos\theta + 1.5$

$3x\cos\theta = 1.5$

$x\cos\theta = 0.5 \rightarrow$ ①.

② L.O.C.O.M in the Vertical plane

$+\uparrow$ $0 + 0 = (3)(x\sin\theta) + (2)(-0.8\sin 20°)$

$3x\sin\theta = (2)(0.8\sin 20°)$

$3x\sin\theta = 0.55$

$x\sin\theta = 0.18 \rightarrow$ ②

To find θ eq② ÷ eq①.

$\frac{x\sin\theta}{x\cos\theta} = \frac{0.18}{0.5}$

$\tan\theta = \frac{0.18}{0.5}$ ∴ $\theta = 20°$

find x by substituting in eq① or ②

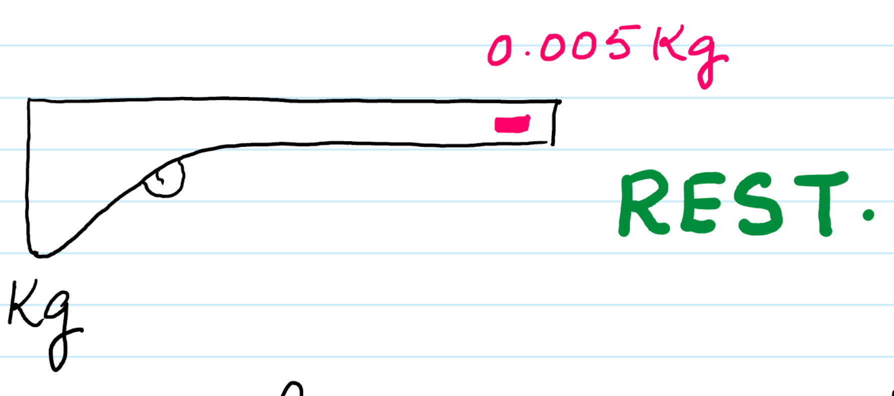
$x\cos\theta = 0.5$ from ①

$x\cos 20° = 0.5$

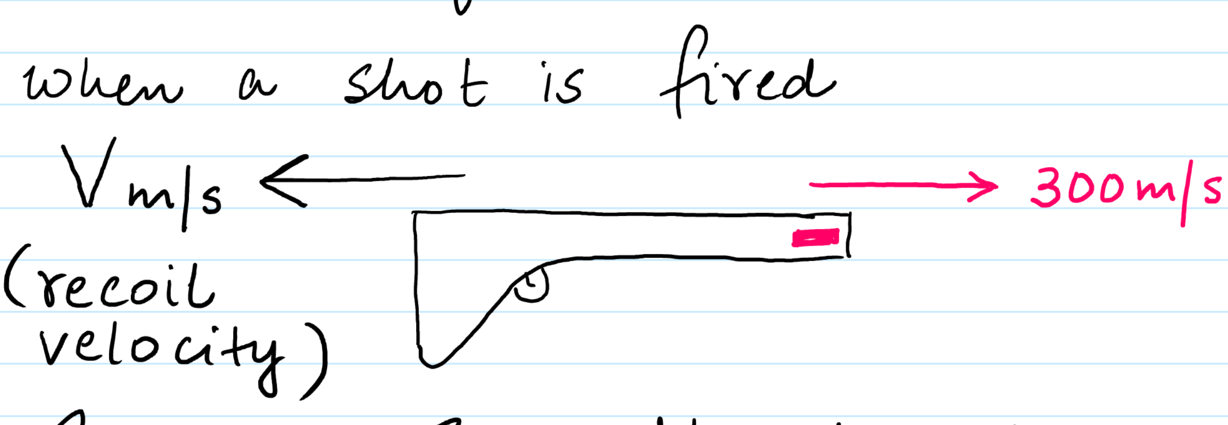
$x = 0.53$ m/s

How to apply Principle of Conservation of momentum in situations where the initial momentum of the system is Zero.

Revolver with a bullet in it, initially both at REST.



when a shot is fired



law of Conservation of momentum

Reason :- Since the initial momentum of the system is ZERO ∴ for L.O.C.O.M to be valid, the final momentum of the system must also remain ZERO.

This is only possible if the 2 bodies have equal momentum in the opposite direction so that they cancel out the effect of each other ∴ when bullet goes forward, the gun must recoil backwards with equal momentum

Cal. the Recoil velocity of the Gun?

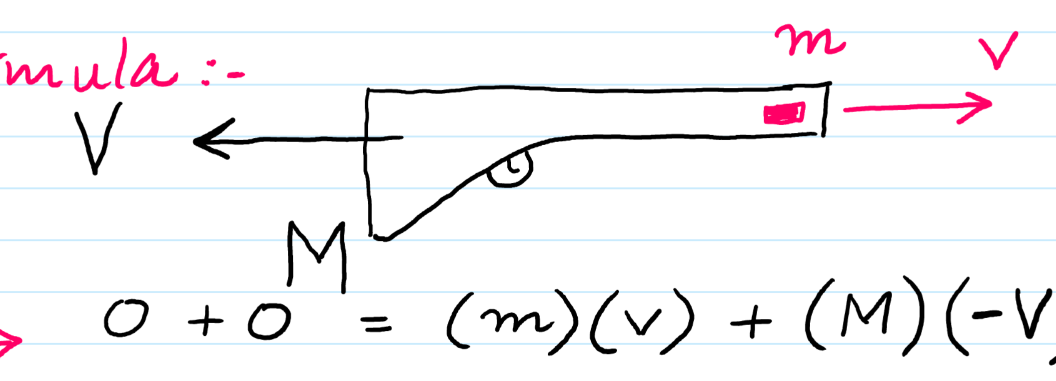
Principle of Conservation of momentum

$\xrightarrow{+}$ $0 + 0 = (0.005)(300) + (2)(-V)$

$(2)(V) = (0.005)(300)$ method ①

$V = 0.75$ m/s.

Gen. formula :-



$\xrightarrow{+}$ $0 + 0 = (m)(v) + (M)(-V)$

$(M)(V) = (m)(v)$

$MV = mv$ method ②

$(2)(V) = (0.005)(300)$

$V = 0.75$ m/s

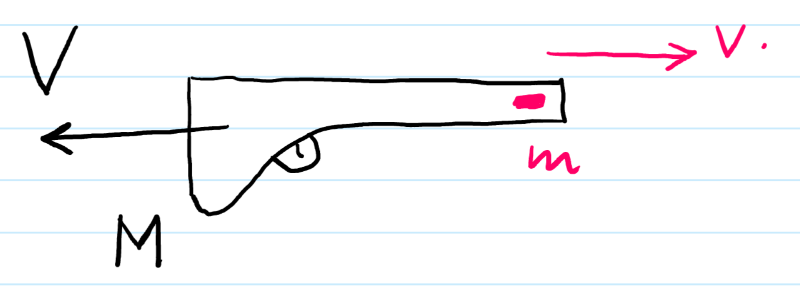
The above working can also be done using ratio method.

According to $p = mv$, m and velocity are inversely proportional to each other hence

$M = 2kg$ $m = 0.005$ kg. method ③

Since mass of gun is "400 times" heavier than the mass of bullet ∴ due to inverse relationship, the velocity of Gun must be "400 times" Lesser than the velocity of bullet hence velocity of gun $= \frac{300}{400} = 0.75$ m/s

Example



Show that

(i) $\frac{M}{m} = \frac{v}{V}$

$\xrightarrow{+}$ $0 + 0 = (m)(v) + (M)(-V)$

$MV = mv$

cross multiply.

$\frac{M}{m} = \frac{v}{V}$ shown.

(ii) Show that

$\frac{K.E \text{ of gun}}{K.E \text{ of bullet}} = \frac{V}{v}$

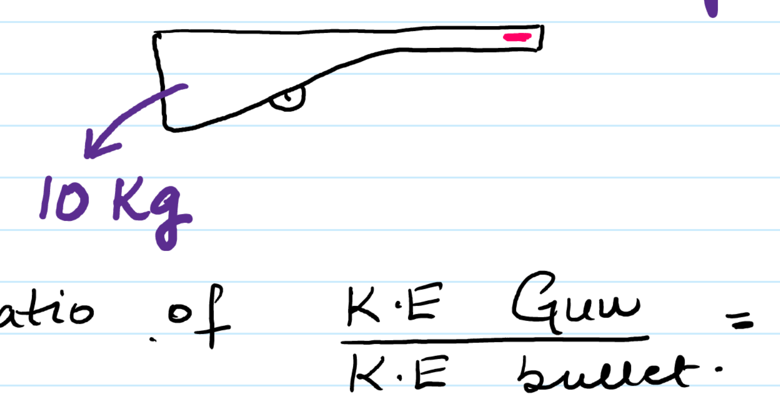
$\frac{K.E\,(Gun) = \frac{1}{2}MV^2}{K.E\,(bullet) = \frac{1}{2}m \cdot v^2}$

$\frac{K.E\,(Gun) = \frac{1}{2} \cdot MV \cdot V}{K.E\,(bullet) = \frac{1}{2} \cdot mv \cdot v}$ Since $M.V = mv$ hence cancels.

$\frac{K.E\,(Gun)}{K.E\,(bullet)} = \frac{V}{v}$ shown.

Conclude :- The ratio of K.E of the 2 bodies actually depends on the ratio of their velocities.

Example



Q Cal. ratio of $\frac{K.E \text{ Gun}}{K.E \text{ bullet}} = ??$

Step ① $\frac{M \text{ gun}}{M \text{ bullet}} = \frac{10}{0.1} = \frac{100}{1}$

Step ② Since $m \propto \frac{1}{v}$

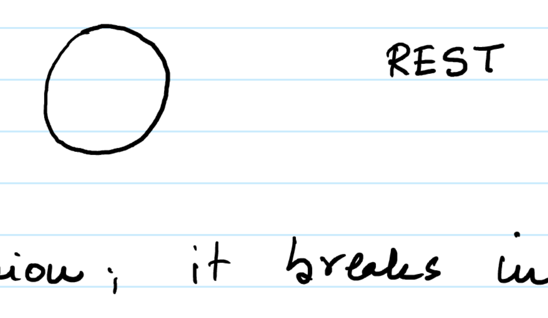
$\frac{V \text{ gun}}{V \text{ bullet}} = \frac{1}{100}$

Step ③ Since ratio of K.E depends on ratio of velocity ∴

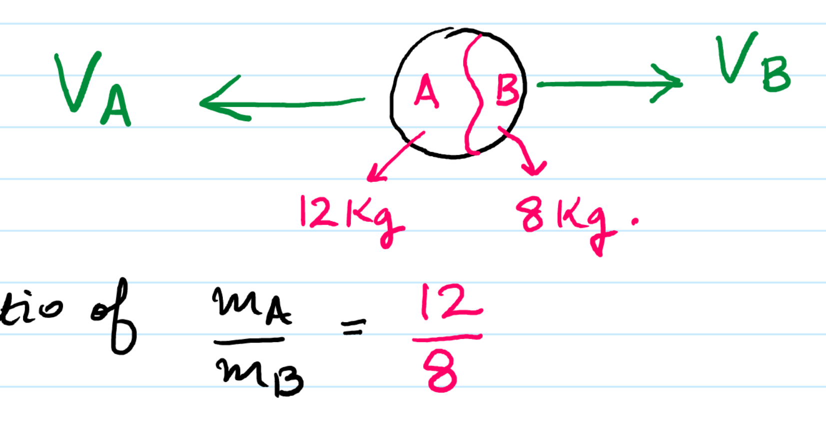
$\frac{K.E \text{ gun}}{K.E \text{ bullet}} = \frac{1}{100}$

Other examples where Initial momentum is Zero

① "Bomb before explosion"



after explosion, it breaks into fragments.



Ratio of $\frac{m_A}{m_B} = \frac{12}{8}$

Ratio of $\frac{V_A}{V_B} = \frac{8}{12}$

Ratio of $\frac{K.E\,(A)}{K.E\,(B)} = \frac{8}{12}$

Total K.E is given as "E"

find fraction of K.E possessed by A $\frac{8}{8+12}E$

find fraction of K.E possessed by B $\frac{12}{8+12}E$