

**39. Transformations**

- use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations
- identify and give precise descriptions of transformations connecting given figures
- describe transformations using coordinates and matrices

If  $M(a) = b$  and  $R(b) = c$  the notation  $RM(a) = c$  will be used.

Invariants under these transformations may be assumed.

Singular matrices are excluded.

Transformations – references removed to shear and stretching.

Notes by Sarwar Khan

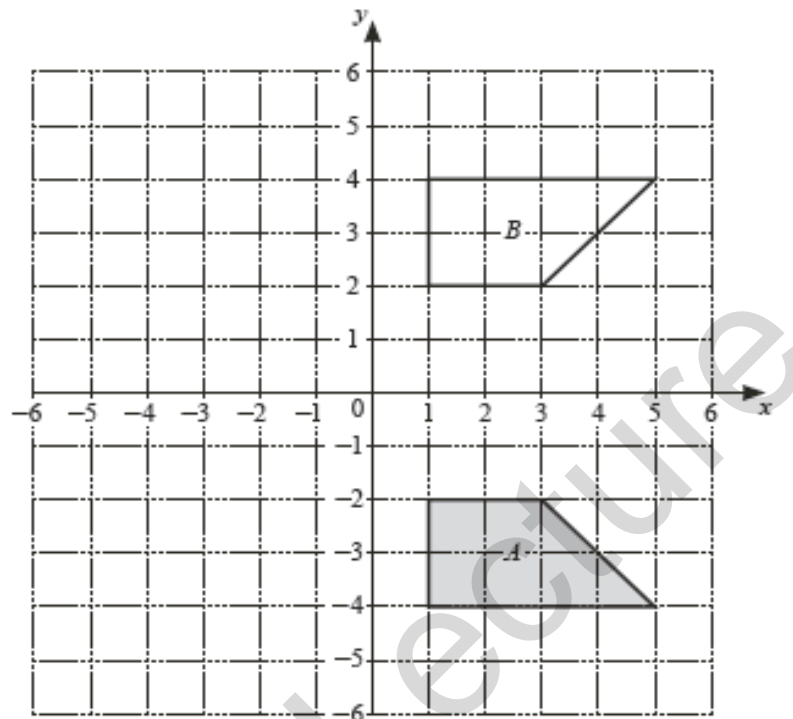
[https://drive.google.com/open?id=1kOOY781PhLBtHHaLzBdNOV1ozqC\\_jYCB](https://drive.google.com/open?id=1kOOY781PhLBtHHaLzBdNOV1ozqC_jYCB)

Transformation playlist(very helpful for understanding basics)

<https://www.youtube.com/watch?v=P1V0o7BxShk&list=PLSunMkRHiawC2PvT03UDLjL2Mcldorz7C>

1

M/J19/11/21



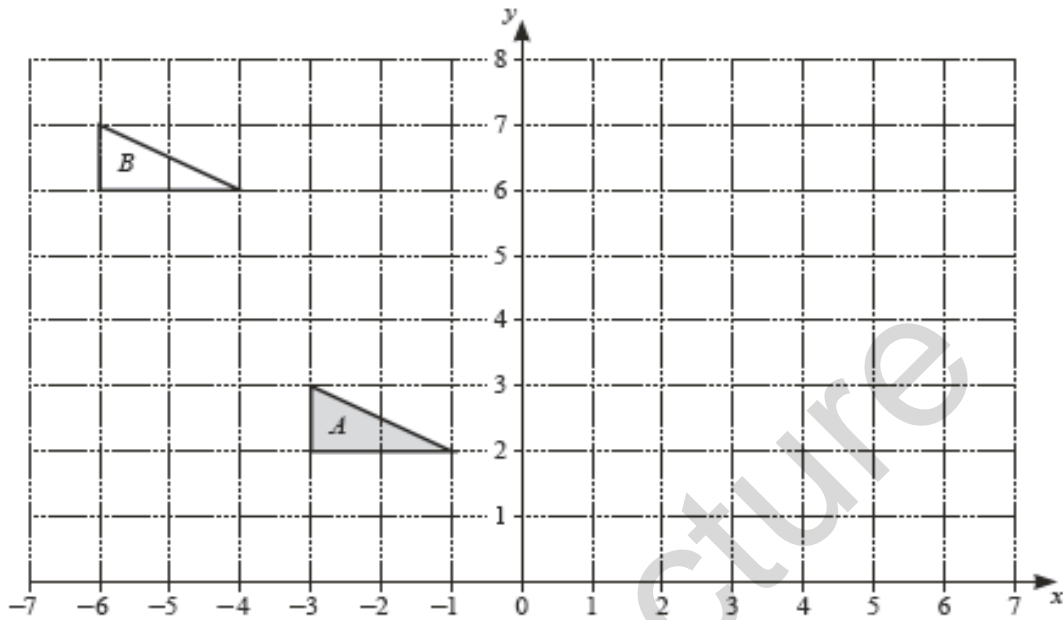
Shapes  $A$  and  $B$  are drawn on the grid.

- (a) Draw the image of shape  $A$  after a translation with vector  $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ . [1]
- (b) Draw the image of shape  $A$  after an enlargement with scale factor  $-\frac{1}{2}$  and centre  $(1, 0)$ . [2]
- (c) Shape  $A$  is mapped onto shape  $B$  by the single transformation  $P$ .
- (i) Describe fully the transformation  $P$ . [2]
- .....
- (ii) Find the matrix representing transformation  $P$ .

$$\begin{pmatrix} & \\ & \end{pmatrix} \quad [1]$$

2

M/J19/12/16



Triangle  $A$  and triangle  $B$  are drawn on the grid.

- (a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

.....  
.....

[2]

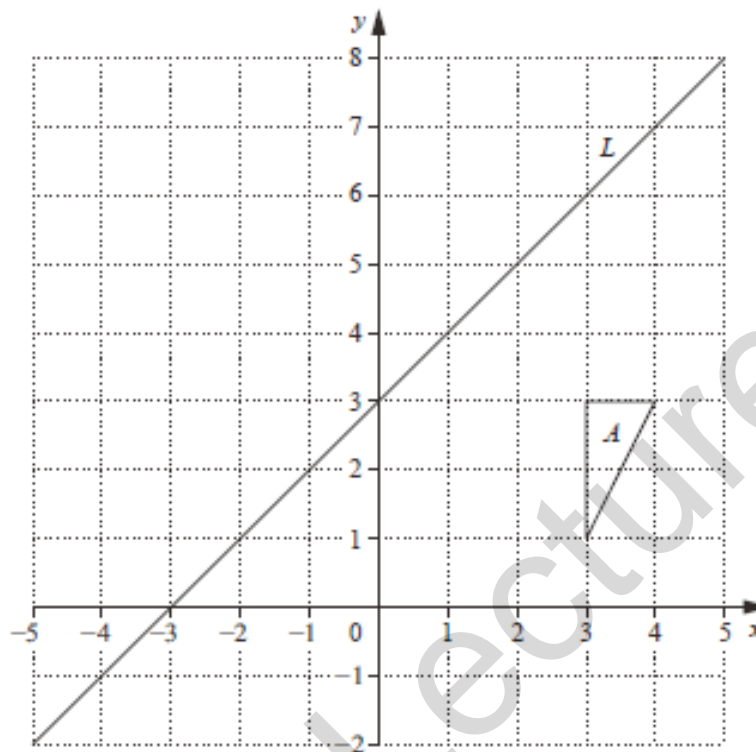
- (b) Triangle  $A$  is mapped onto triangle  $C$  by an enlargement with centre  $(0, 3)$  and scale factor  $-2$ .

On the grid, draw triangle  $C$ .

[2]

3

SP18/02/3



The grid shows triangle  $A$  and line  $L$ .

- (a) (i) Triangle  $A$  is mapped onto triangle  $B$  by a reflection in line  $L$ .

Draw and label triangle  $B$  on the grid.

[2]

- (ii) Triangle  $A$  is mapped onto triangle  $C$  by a clockwise rotation of  $90^\circ$ , centre  $(0, 3)$ .

Draw and label triangle  $C$  on the grid.

[2]

- (iii) Triangle  $C$  is mapped onto triangle  $D$  by a reflection in line  $L$ .

Describe the single transformation that maps triangle  $B$  onto triangle  $D$ .

Answer .....

..... [3]

- (b) The line  $P$  is parallel to line  $L$  and passes through the point  $(0, 5)$ .

Find the equation of line  $P$ .

Answer ..... [2]

- (c) The line  $R$  is perpendicular to line  $L$  and passes through the origin  $(0, 0)$ .

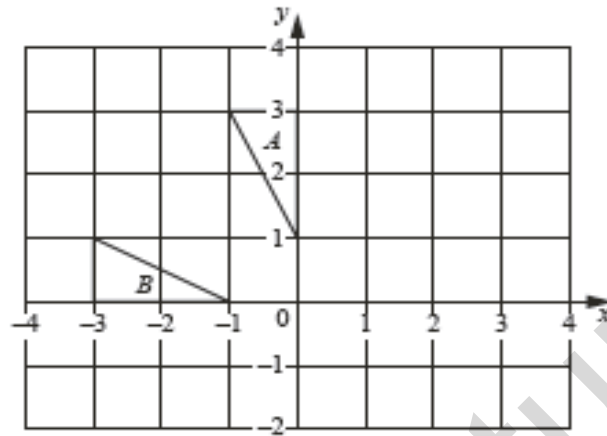
Find the equation of line  $R$ .

Answer ..... [1]

Mega Lecture

4

O/N18/11/22



The diagram shows triangles  $A$  and  $B$ .

- (a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

.....  
 ..... [2]

- (b) Triangle  $A$  is mapped onto triangle  $C$  by a rotation, through  $90^\circ$  clockwise, centre  $(0, 0)$ .

Draw, and label, triangle  $C$  on the diagram. [2]

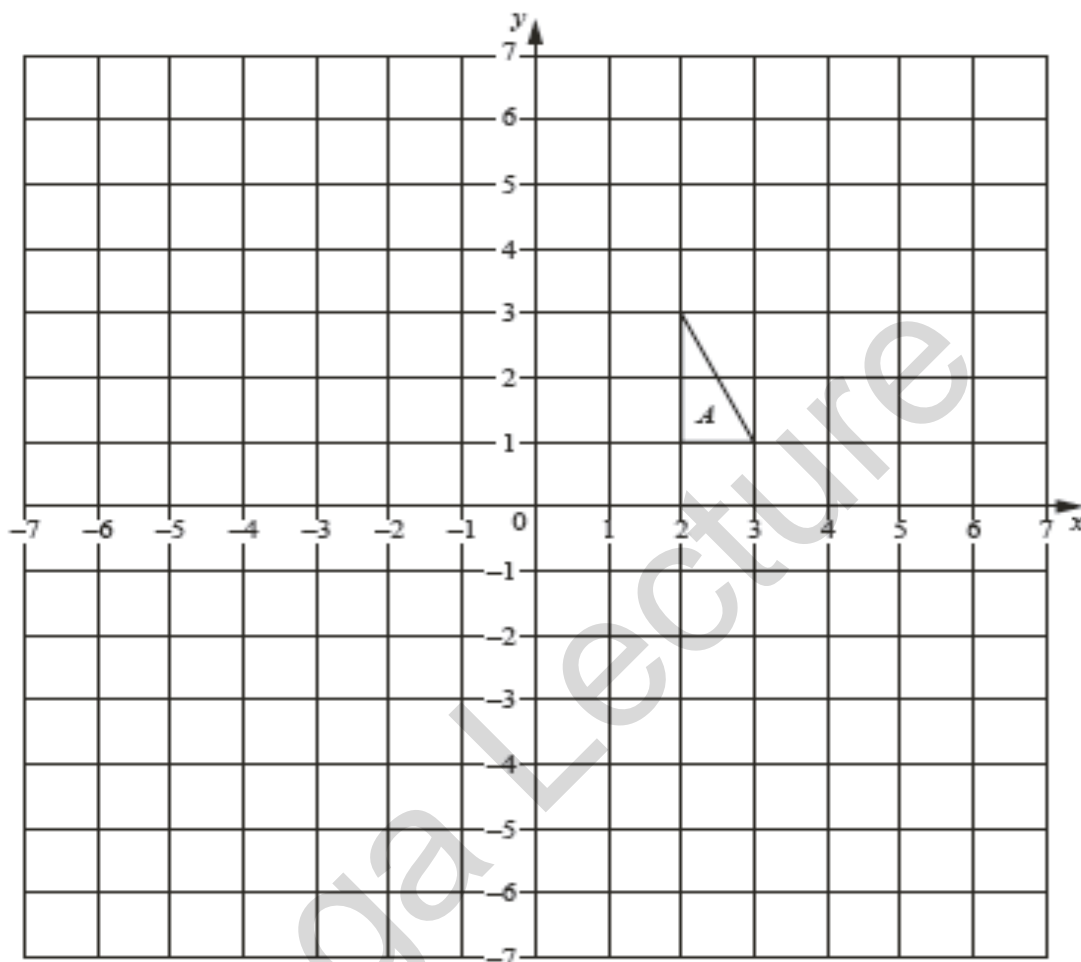
- (c) Triangle  $B$  is mapped onto triangle  $C$  by the transformation  $T$ .

Find the matrix that represents the transformation  $T$ .

Answer  $\begin{pmatrix} & \\ & \end{pmatrix}$  [1]

5

O/N18/22/7



Triangle  $A$  is drawn on the grid.

(a) Transformation  $P$  is represented by the matrix  $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ .

$P$  maps triangle  $A$  onto triangle  $B$ .

(i) Draw and label triangle  $B$ .

[2]

(ii) Describe fully the single transformation P.

.....  
..... [2]

(iii) Write down the ratio area of triangle A : area of triangle B.

Answer ..... : ..... [1]

(b) Transformation Q is represented by the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

Q maps triangle B onto triangle C.

Draw and label triangle C.

[2]

(c) Transformation Y is represented by the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ .

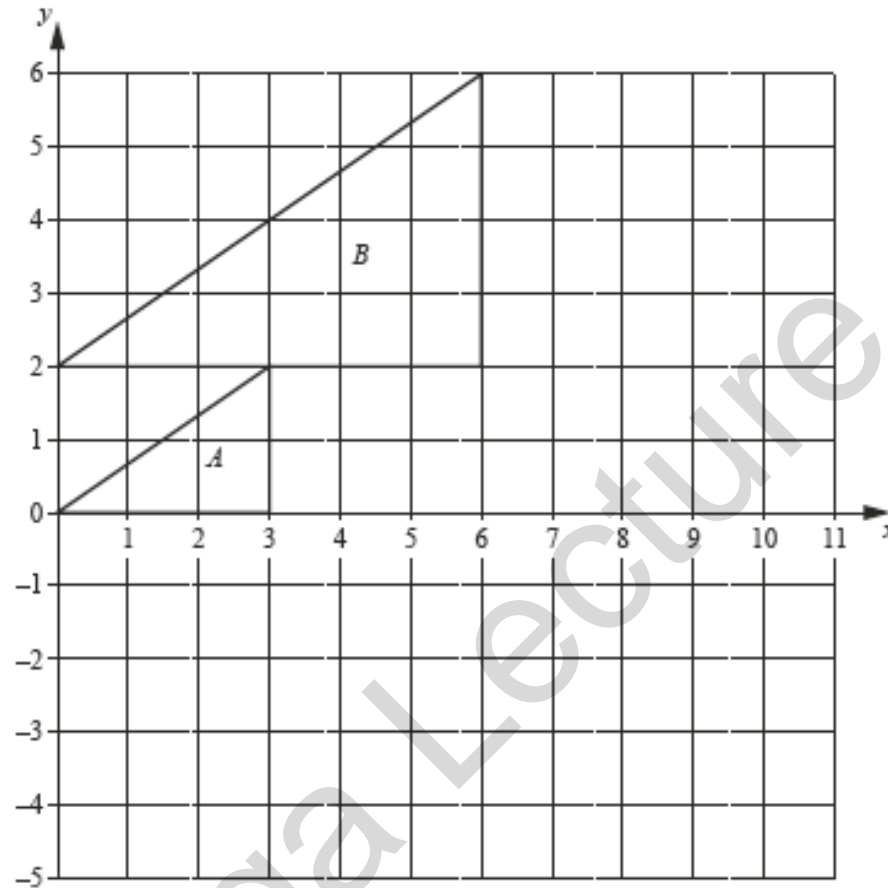
Y maps triangle A onto triangle D.

Find the matrix that represents the transformation that maps triangle D onto triangle A.

Answer  $\begin{pmatrix} & \\ & \end{pmatrix}$  [2]



6



Triangle  $A$  is mapped onto triangle  $B$  by a translation, followed by an enlargement with centre  $(10, -4)$ .  
 The translation maps triangle  $A$  onto triangle  $C$ .  
 The enlargement maps triangle  $C$  onto triangle  $B$ .

(a) Write down the scale factor of the enlargement.

Answer ..... [1]

(b) Draw triangle  $C$  on the grid.

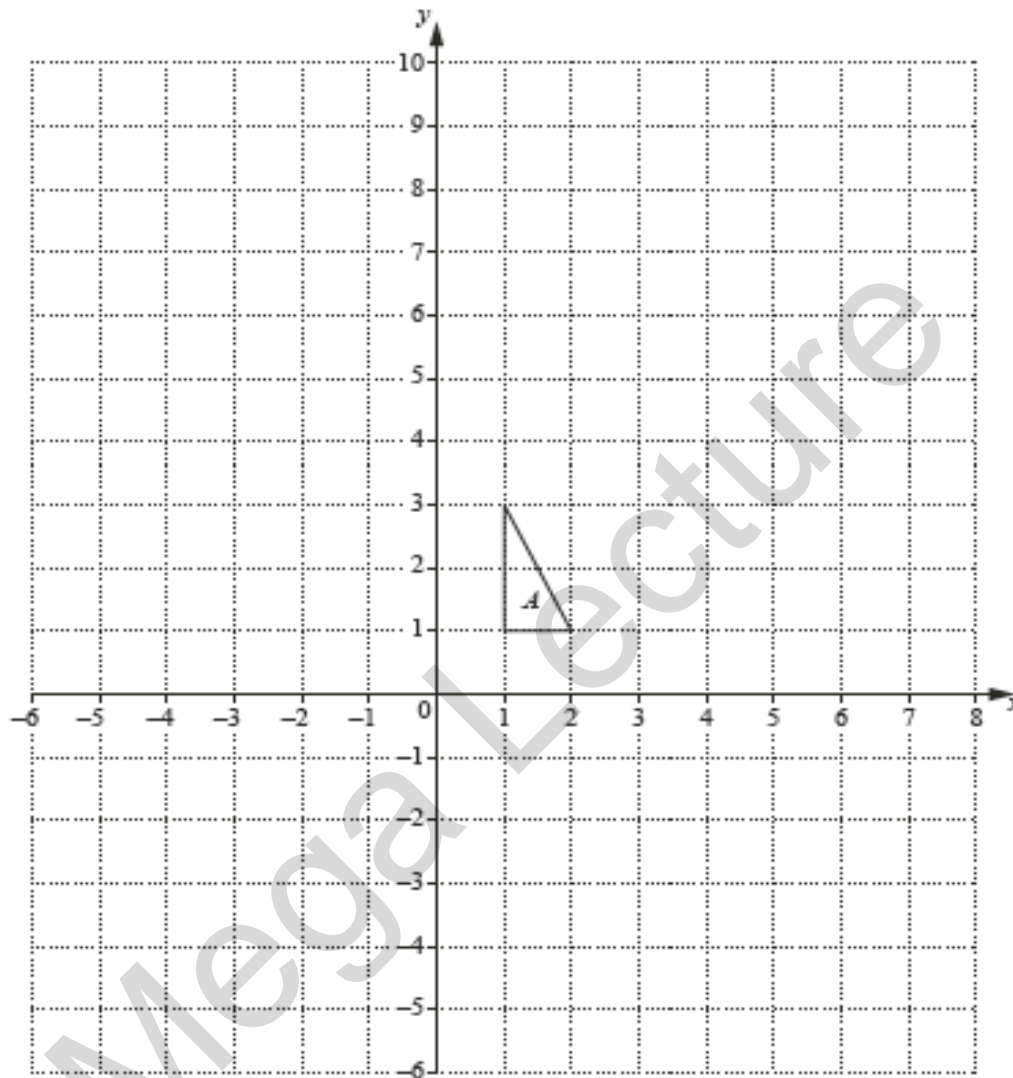
[2]

(c) Find the column vector that represents the translation that maps triangle  $A$  onto triangle  $C$ .

Answer  $\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$  [1]

- 7 (a) Triangle  $A$  is shown on the grid.

O/N17/21/10(a)



- (i) Triangle  $A$  is mapped onto triangle  $B$  by a rotation of  $180^\circ$  about point  $(2, -1)$ .

Draw and label triangle  $B$  on the grid.

[2]

- (ii) Triangle  $A$  is mapped onto triangle  $C$  by the transformation represented by the matrix  $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ .

Draw and label triangle  $C$  on the grid.

[2]

(iii) Write down the matrix that represents the transformation that maps triangle  $C$  onto triangle  $A$ .

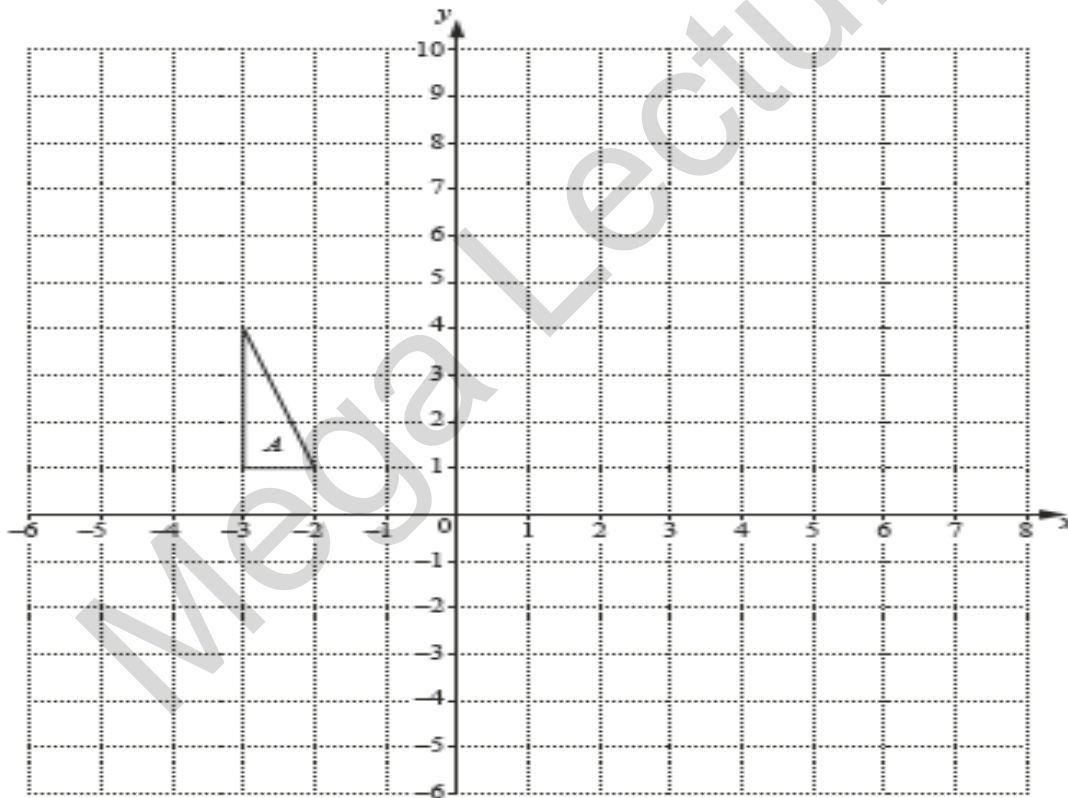
Answer  $\begin{pmatrix} & \\ & \end{pmatrix}$  [1]

(iv) Describe fully the single transformation that maps triangle  $C$  onto triangle  $B$ .

.....  
 ..... [3]

8 (a) Triangle  $A$  is shown on the grid.

O/N17/22/4(a)

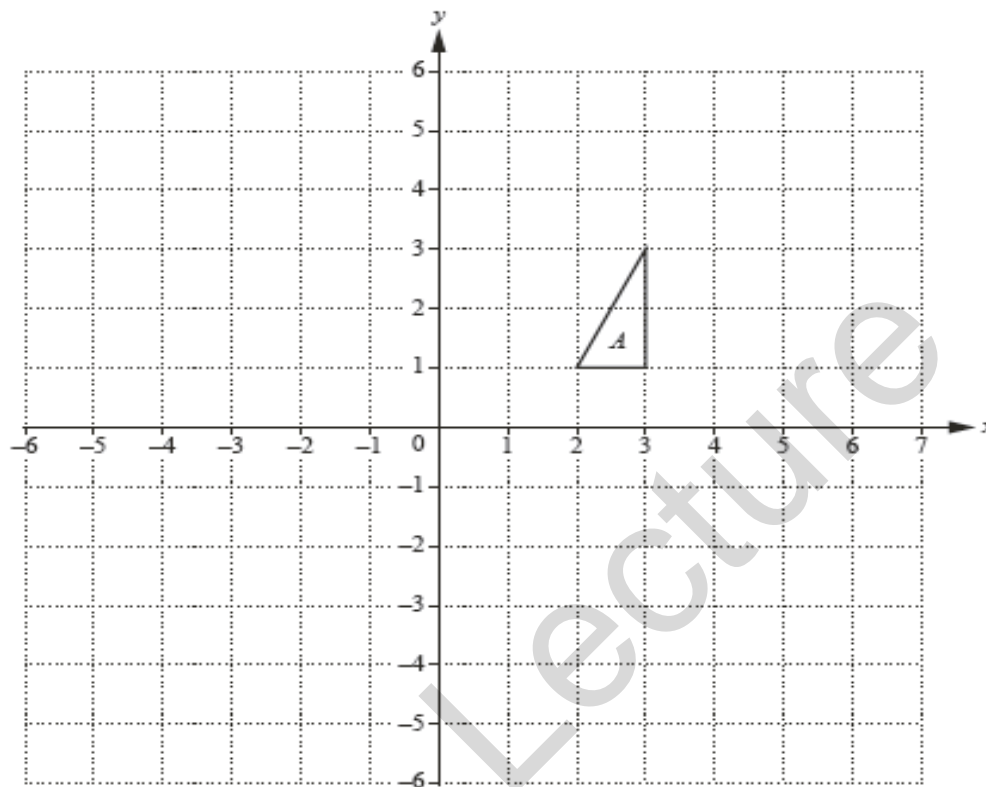


(i) Triangle  $A$  is mapped onto triangle  $B$  by a translation of  $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$ .  
 Draw and label triangle  $B$  on the grid. [2]

(ii) Triangle  $A$  is mapped onto triangle  $C$  by an enlargement scale factor  $-2$ , centre  $(-1, 2)$ .  
 Draw and label triangle  $C$  on the grid. [2]

9 The diagram shows triangle  $A$ .

M/J17/11/17(a)



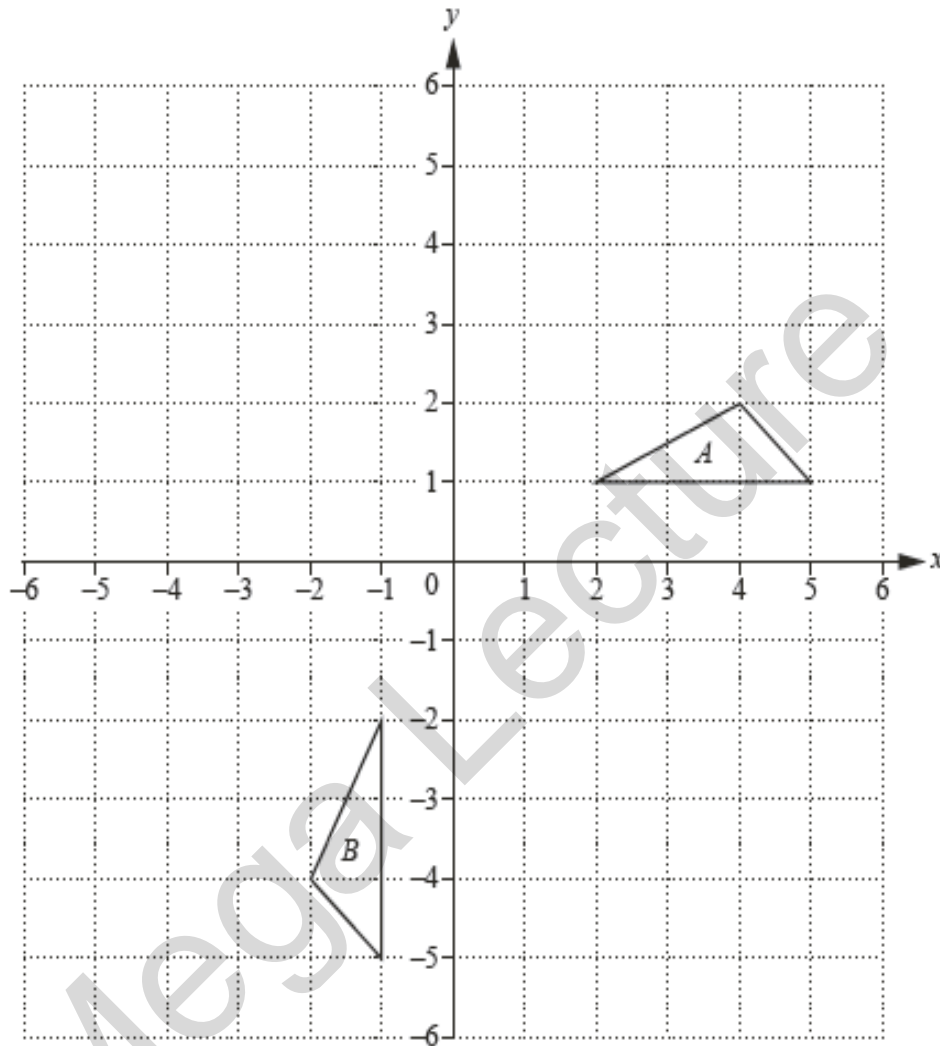
(a) Triangle  $B$  is the image of triangle  $A$  after reflection in the line  $y = -1$ .

Draw and label triangle  $B$  on the diagram.

[1]

10 The diagram shows triangles  $A$  and  $B$ .

M/J17/12/15



(a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

Answer .....

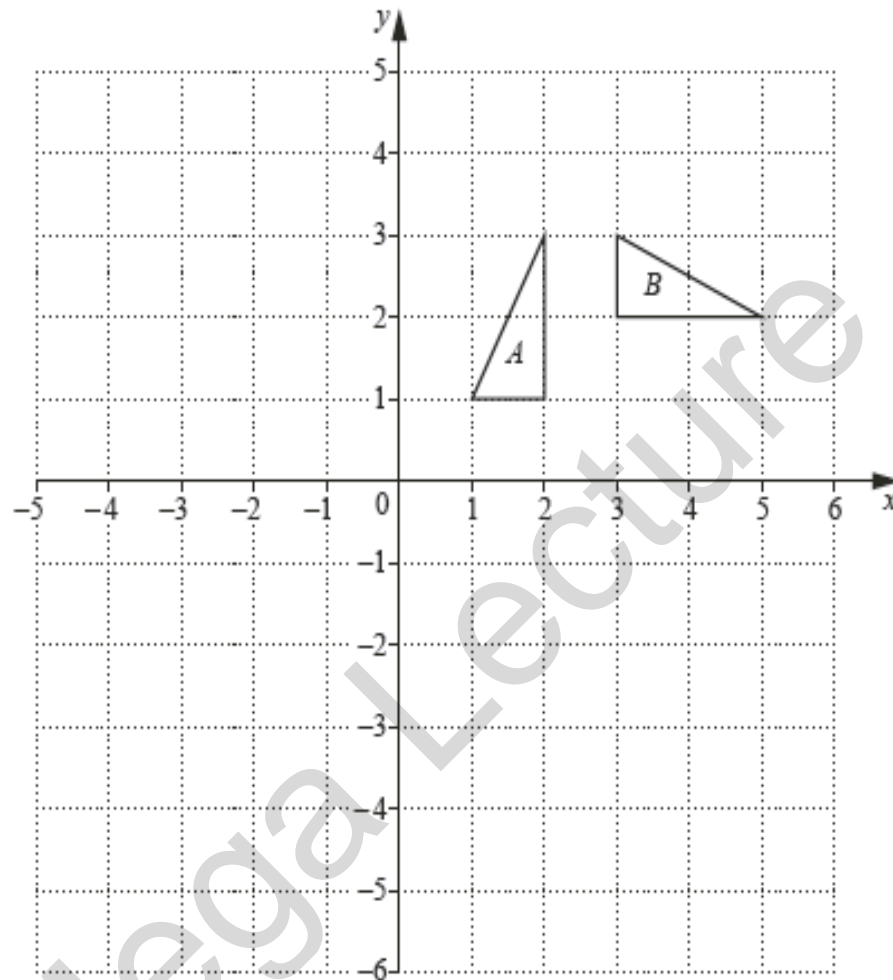
[2]

(b) Triangle  $A$  is mapped onto triangle  $C$  by a rotation,  $90^\circ$  anti-clockwise about the origin.

On the diagram, draw triangle  $C$ .

[2]

11



- (a) Describe the single transformation that maps triangle *A* onto triangle *B*.

Answer .....

..... [2]

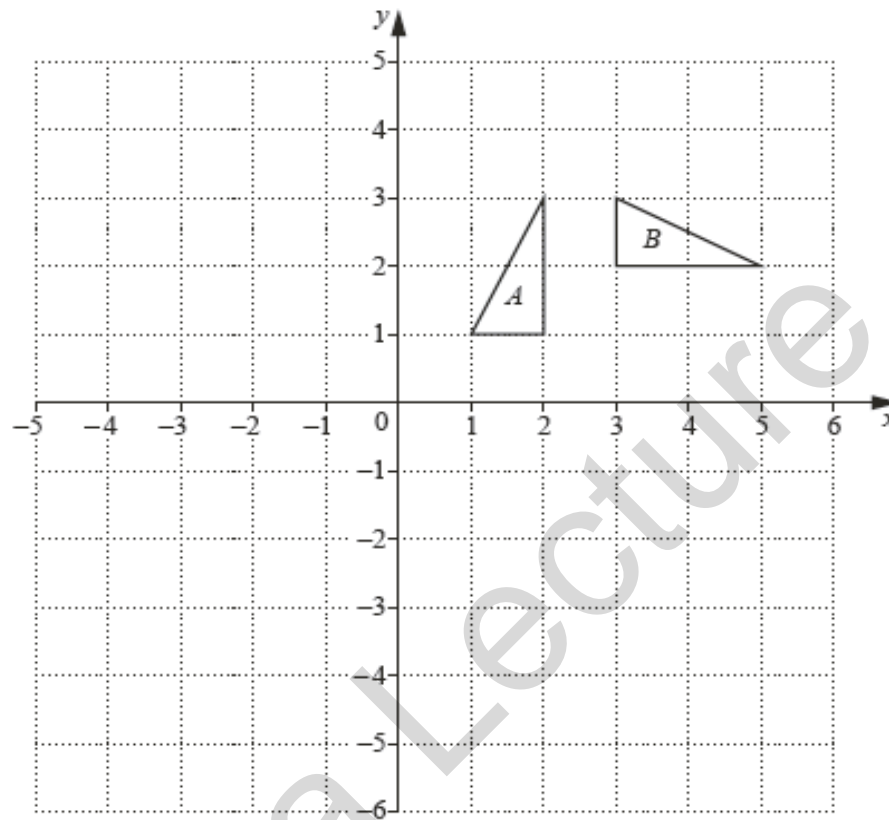
- (b) Triangle *A* is mapped onto triangle *C* by an enlargement, centre (0, 2) and scale factor  $-2$ .

Draw, and label, triangle *C* on the diagram.

[2]

11

ON16/11/15



- (a) Describe the single transformation that maps triangle *A* onto triangle *B*.

Answer .....

..... [2]

- (b) Triangle *A* is mapped onto triangle *C* by an enlargement, centre (0, 2) and scale factor  $-2$ .

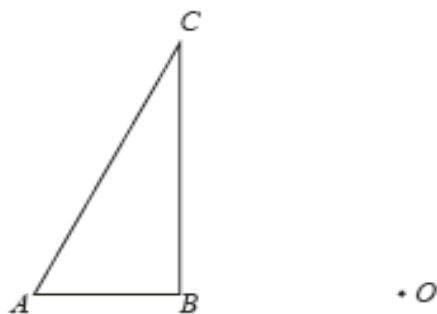
Draw, and label, triangle *C* on the diagram.

[2]

12 Triangle  $ABC$  is mapped onto triangle  $A'B'C'$  by a rotation, centre  $O$ , through  $110^\circ$  clockwise.

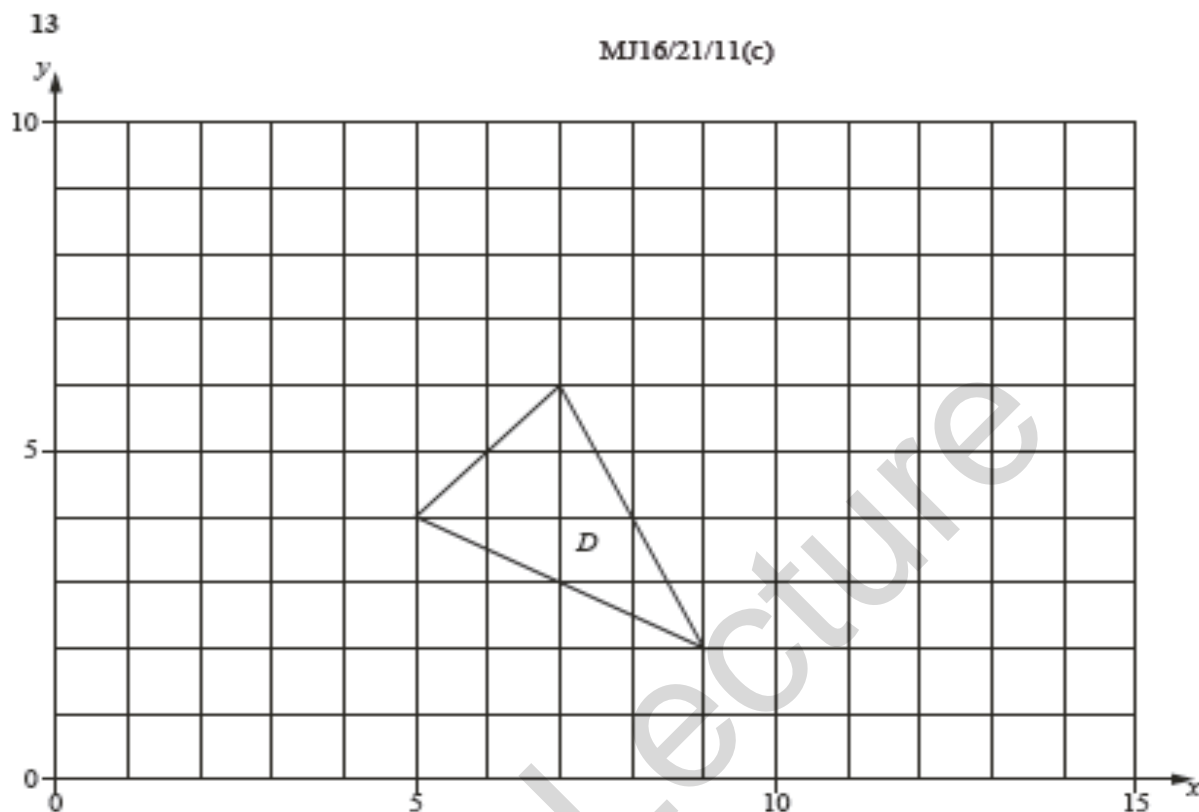
Draw and label triangle  $A'B'C'$ .

O/N16/12/13



[3]





The diagram shows triangle  $D$ .

- (i) An enlargement with centre  $(5, 4)$ , scale factor 2, maps triangle  $D$  onto triangle  $E$ .

Draw and label triangle  $E$ .

[2]

- (ii) An enlargement with centre  $(5, 4)$ , scale factor 0.5, maps triangle  $D$  onto triangle  $F$ .

Draw and label triangle  $F$ .

[1]

- (iii) Triangle  $G$  has vertices  $(5, 4)$ ,  $(4, 3)$  and  $(3, 5)$ .

Triangle  $F$  can be mapped onto triangle  $G$  using a single enlargement.

Triangle  $F$  can also be mapped onto triangle  $G$  using a different single transformation  $T$ .

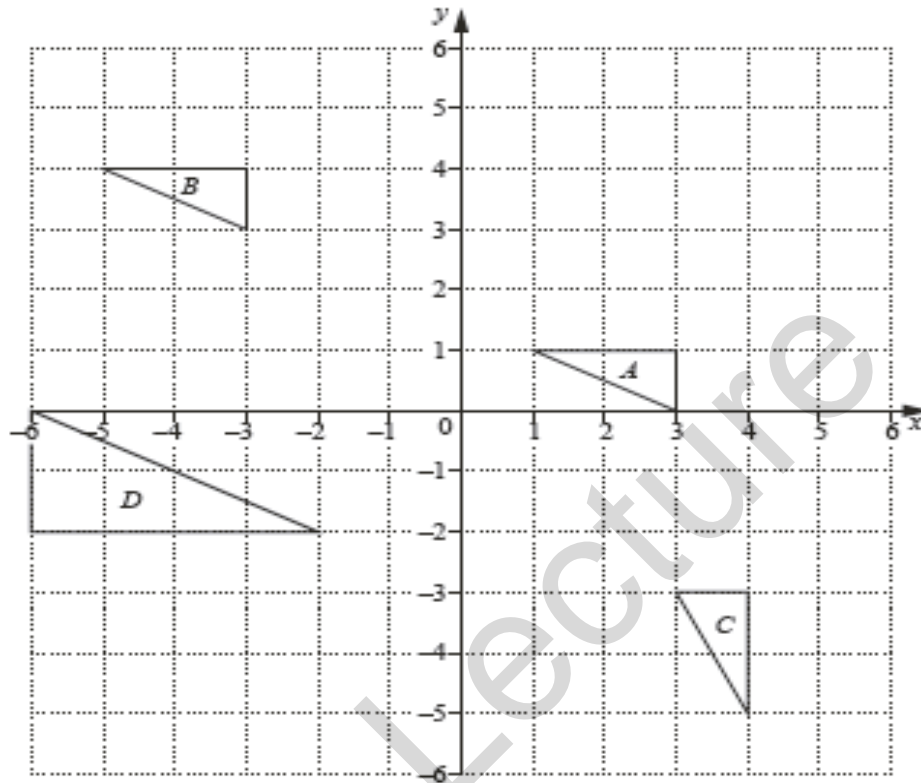
Describe fully the single transformation  $T$ .

Answer .....

.....

..... [3]

14 Triangles  $A$ ,  $B$ ,  $C$  and  $D$  are drawn on a centimetre square grid. M/J16/22/10



(a) The perimeter of triangle  $A$  is  $(a + \sqrt{b})$  cm, where  $a$  and  $b$  are integers.

Find  $a$  and  $b$ .

Answer  $a = \dots\dots\dots b = \dots\dots\dots$  [2]

(b) Triangle  $A$  is mapped onto triangle  $B$  by the translation  $T$ .

Write down the column vector that represents  $T$ .

Answer  $\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$  [1]

(c) Describe fully the single transformation that maps triangle  $B$  onto triangle  $C$ .

Answer .....  
 ..... [2]

- (d) Describe fully the single transformation that maps triangle  $B$  onto triangle  $D$ .

Answer .....

..... [3]

- (e) Write down the matrix that represents the transformation which maps triangle  $D$  onto triangle  $A$ .

Answer [1]

- (f) The transformation  $V$  is a reflection in the line  $y = 0$ .  
The transformation  $W$  is a rotation  $90^\circ$  clockwise about  $(0, 0)$ .  
The single transformation  $X$  is equivalent to the transformation  $V$  followed by the transformation  $W$ .

- (i) The point  $(g, h)$  is mapped onto the point  $P$  by the transformation  $X$ .

Find the coordinates of  $P$ .

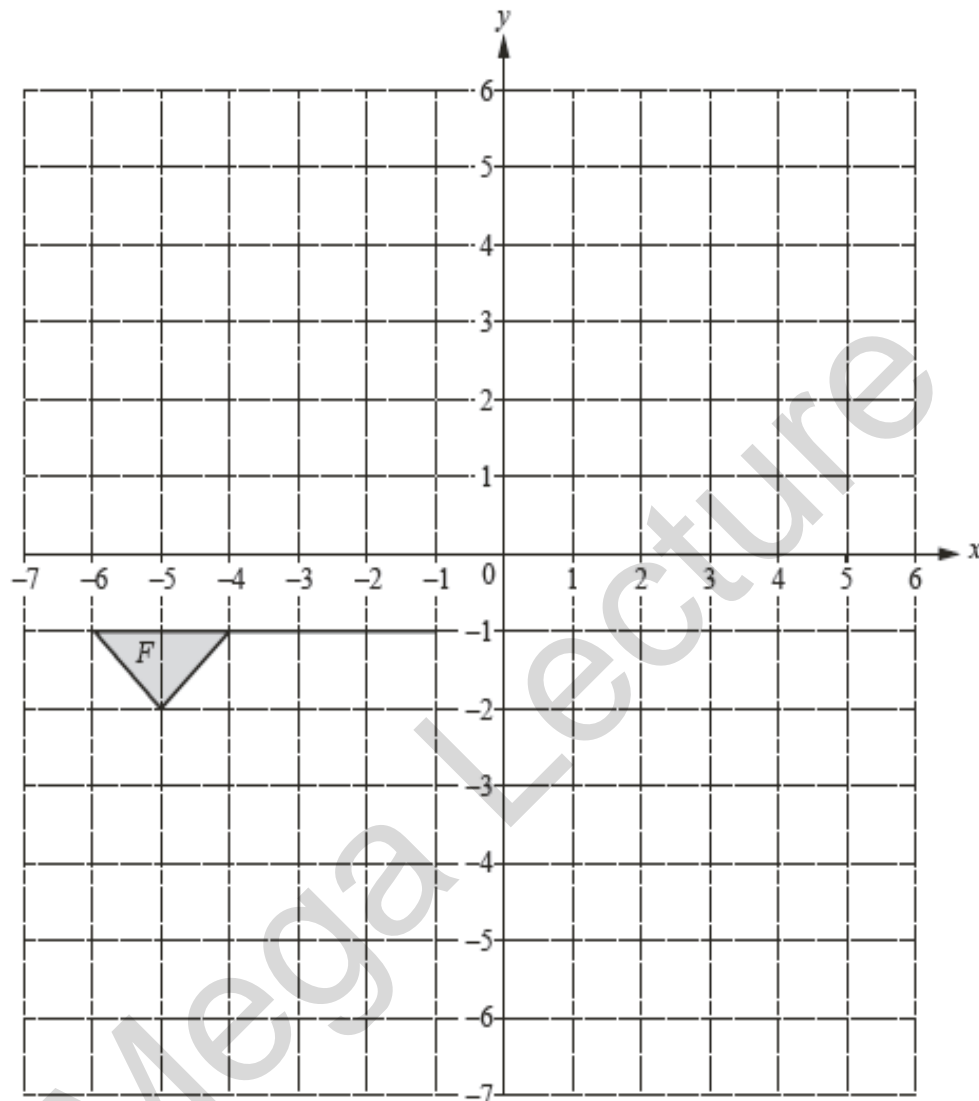
Answer (....., .....) [1]

- (ii) Describe fully the single transformation  $X$ .

Answer .....

..... [2]

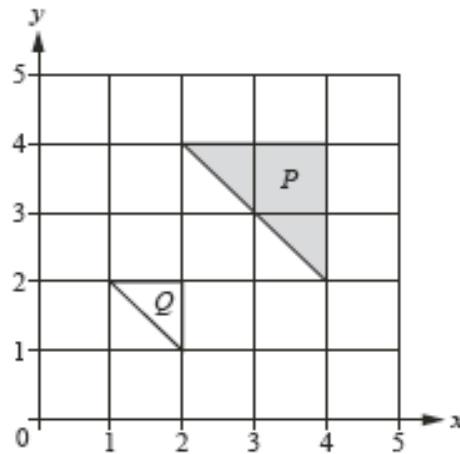
15 (a)



Draw the image of

- (i) flag  $F$  after translation by the vector  $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ , [2]
- (ii) flag  $F$  after rotation through  $180^\circ$  about  $(-2, 0)$ , [2]
- (iii) flag  $F$  after reflection in the line  $y = x$ . [2]

(b)



(i) Describe fully the single transformation that maps triangle  $P$  onto triangle  $Q$ .

.....  
 .....

[3]

(ii) Find the matrix that represents this transformation

$\begin{pmatrix} & \\ & \end{pmatrix}$  [2]

(c) The point  $A$  is translated to the point  $B$  by the vector  $\begin{pmatrix} 4u \\ 3u \end{pmatrix}$ .

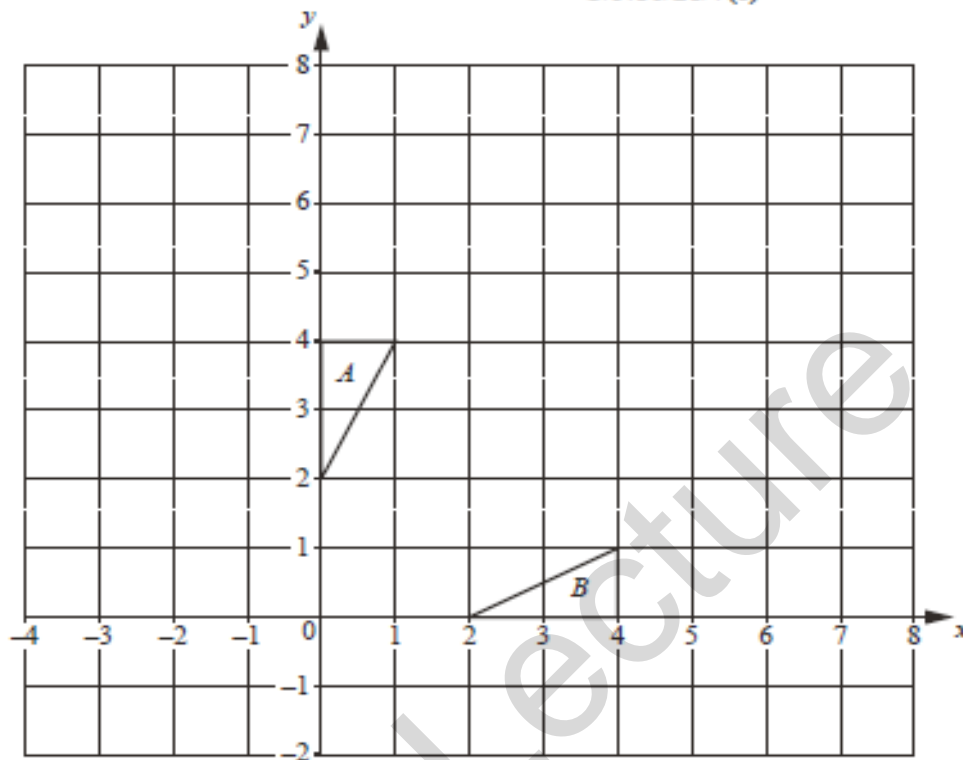
$|\vec{AB}| = 12.5$

Find  $u$ .

$u = \dots\dots\dots$  [3]

(16(a))

O/N16/21/7(c)



The diagram shows triangle *A* and triangle *B*.

- (i) Triangle *A* is mapped onto triangle *C* by the translation *P* with vector  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ .

Draw and label triangle *C*.

[2]

- (ii) Triangle *A* is mapped onto triangle *B* by a reflection *Q*.

Write down the equation of the line of this reflection.

Answer ..... [1]

- (iii) Triangle *C* is mapped onto triangle *D* by reflection *Q*.

Describe fully the single transformation that maps triangle *B* onto triangle *D*.

Answer ..... [2]

(iv) Transformation R is a reflection in the line  $y = 0$ .

$$RQ(A) = E.$$

(a) Find the coordinates of the vertices of triangle E.

*Answer* ..... [1]

(b) Describe fully the single transformation that maps triangle A onto triangle E.

*Answer* .....

..... [2]

(c) Find the matrix which represents the transformation that maps triangle A onto triangle E.

*Answer* ..... [1]

Mega Lecture

Marking Scheme

|           |  |   |  |
|-----------|--|---|--|
| 1(a)      | Trapezium drawn with vertices $(-3,-1)$ $(-1,-1)$ $(1,-3)$ $(-3,-3)$ | 1 |  |
| 1(b)      | Trapezium drawn with vertices $(1,1)$ $(1,2)$ $(0,1)$ $(-1,2)$       | 2 | <b>B1</b> for correct size and orientation, wrong centre or for correct enlargement scale factor $\frac{1}{2}$ centre $(1, 0)$ . |
| 1(c)(i)   | Reflection in the $x$ -axis or $y = 0$                               | 2 | <b>B1</b> for either   |
| 2(a)      | Translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$                  | 2 | <b>B1</b> for translation<br><b>B1</b> for $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$   |
| 2(b)      | Correct enlargement, vertices $(2, 5)$ , $(6, 5)$ , $(6, 3)$         | 2 | <b>B1</b> for correct size and orientation, incorrect position<br>or for enlargement scale factor 2, centre $(0, 3)$             |
| 3(a)(i)   | Correct triangle   | 2 | <b>B1</b> for two correct vertices or triangle correct size and orientation  |
| 3(a)(ii)  | Correct triangle   | 2 | <b>B1</b> for two correct vertices or triangle correct size and orientation  |
| 3(a)(iii) | Complete description www   | 3 | <b>B1</b> for Rotation<br><b>B1</b> for either 90 anticlockwise or centre $(0, 3)$   |
| 3(b)      | $y = x + 5$  | 2 | <b>B1</b> for either $y = x + k$ , $k \neq 5$<br>or for $y = mx + 5$ , $m \neq 0$ or 1   |
| 3(c)      | $y = -x$   | 1 |  |



|      |   |   |  |
|------|---|---|--|
| 4(a) | Reflection<br>and $y = -x$ oe                   | 2 | B1 for either  |
| 4(b) | Triangle with vertices<br>(1,0), (3,0), (3,1)   | 2 | B1 for 90° clockwise rotation with wrong centre, or for the triangle with vertices (-1,0), (-3,0), (-3,-1) |
| 4(c) | $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ | 1 |  |

|           |   |   |   |
|-----------|---|---|---|
| 5(a)(i)   | Triangle B at (-4, -2), (-6, -2), (-4, -6)  | 2 | B1 for two vertices correct or two correct pairs of coordinates soi or correct size and orientation but wrong position  |
| 5(a)(ii)  | Enlargement, centre (0, 0) oe, scale factor -2  | 2 | B1 for enlargement  |
| 5(a)(iii) | 1 : 4 oe  | 1 |   |
| 5(b)      | Triangle C at (-4, 2), (-6, 2), (-4, 6)   | 2 | FT reflection of <i>their</i> triangle B in x-axis<br>B1FT for two vertices correct   |
| 5(c)      | $\frac{1}{3}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ isw | 2 | B1 for $k\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ oe with $k \neq \frac{1}{3}$<br>or for $\frac{1}{3}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$ |

|      |  |   |  |
|------|--|---|--|
| 6(a) | 2  | 1 |  |
| 6(b) | Triangle with vertices<br>(5, -1), (8, -1), (8, 1) | 2 | B1 for two correct vertices, soi<br>or M1 for a line joining (10, -4) to a vertex of triangle B. |
| 6(c) | $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$            | 1 |  |

|           |   |   |   |
|-----------|---|---|---|
| 6(a)(i)   | Triangle $B$ at $(2, -3), (3, -3), (3, -5)$                           | 2 | <b>B1</b> for translation of correct triangle $B$   |
| 6(a)(ii)  | Triangle $C$ at $(3, 3), (3, 9), (6, 3)$                              | 2 | <b>B1</b> for two vertices correct or for<br>$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ oe |
| 6(a)(iii) | $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ oe | 1 |   |
| 6(a)(iv)  | Enlargement<br>Centre $(3, -1.5)$<br>SF $-\frac{1}{3}$                | 3 | <b>B1</b> for each  |

|           |   |   |   |
|-----------|---|---|---|
| 7(a)(i)   | Triangle $B$ at $(2, -3), (3, -3), (3, -5)$                           | 2 | <b>B1</b> for translation of correct triangle $B$   |
| 7(a)(ii)  | Triangle $C$ at $(3, 3), (3, 9), (6, 3)$                              | 2 | <b>B1</b> for two vertices correct or for<br>$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ oe |
| 7(a)(iii) | $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ oe | 1 |   |
| 7(a)(iv)  | Enlargement<br>Centre $(3, -1.5)$<br>SF $-\frac{1}{3}$                | 3 | <b>B1</b> for each  |

|          |   |   |   |
|----------|---|---|---|
| 8(a)(i)  | Triangle $B$ at $(4, -1), (4, -4), (5, -4)$ | 2 | <b>B1</b> For triangle $B$ the correct size and orientation   |
| 8(a)(ii) | Triangle $C$ at $(1, 4), (3, 4), (3, -2)$   | 2 | <b>B1</b> for correct size and orientation, incorrect position<br><b>or</b> for triangle with two vertices correct<br><b>or</b> for triangle at $(-3, 0), (-5, 0), (-5, 6)$ |

|      |   |   |  |
|------|---|---|--|
| 9(a) | <b>B</b> drawn with vertices<br>$(2, -3), (3, -3), (3, -5)$ | 1 |  |
|------|---|---|--|

|               |  |        |  |
|---------------|--|--------|--|
| 10(a)         | Reflection<br>$y = -x$ oe                              | 2      | <b>C1</b> for reflection<br>or for $y = -x$ oe   |
| 10(b)         | Triangle vertices $(-1, 2)$ , $(-1, 5)$ ,<br>$(-2, 4)$ | 2      | <b>C1</b> for correct size and orientation, incorrect<br>position<br>or for $90^\circ$ clockwise rotation about origin   |
| <b>11 (a)</b> | Rotation<br>$90^\circ$ clockwise oe, centre $(3, 1)$   | 1<br>1 | Mark lost if a second transformation is<br>named.  |
| <b>(b)</b>    | vertices: $(-2, 4)$ , $(-4, 0)$ , $(-4, 4)$            | 2 *    | <b>B1</b> for two correct vertices,<br>or for vertices $(2, 0)$ , $(4, 0)$ , $(4, 4)$  |
| <b>12</b>     | Correct triangle                                       | 3*     | Following an attempt at a rotation of<br>$110^\circ$ about $O$ , award<br><b>C2</b> for two correct vertices<br>or <b>C1</b> for one correct vertex.<br><br>If [0] scored then either<br><b>B1</b> for arc(s) of correct radii, centre $O$ ,<br>(from $A$ , $B$ or $C$ );<br>or <b>B1</b> for $AOA'$ or $BOB'$ or $COC' = 110^\circ$ |
| <b>13 (i)</b> | Triangle vertices $(5,4)$ , $(13,0)$ ,<br>$(9,8)$      | 2      | B1 for 2 correct   |
| <b>(ii)</b>   | Triangle $F$ $(5,4)$ , $(7,3)$ , $(6,5)$               | 1      |  |
| <b>(iii)</b>  | Rotation<br>180<br>Centre $(5,4)$                      | 3      | B2 for Rotation with either centre or angle.<br><br>B1 for Rotation.   |

|        |  |   |  |
|--------|--|---|--|
| 14 (a) | $a = 3$ $b = 5$  | 2 | <b>B1</b> for one correct  |
| (b)    | $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$ or $3 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ | 1 |  |
| (c)    | Reflection, $y = x$  | 2 | <b>B1</b> for reflection<br>or <b>B1</b> for $y = x$ only  |
| (d)    | Enlargement, Scale factor $-2$ , centre $(-4, 2)$                                    | 3 | <b>B1</b> for enlargement /<br>negative enlargement<br><b>B1</b> for scale factor $-2$<br><b>B1</b> for centre $(-4, 2)$ |
| (e)    | $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ oe              | 1 |  |

|         |                     |   |  |
|---------|---------------------|---|--|
| (f) (i) | $(-h, -g)$          | 1 |  |
| (ii)    | Reflection $y = -x$ | 2 | <b>B1</b> for reflection<br>or <b>B1</b> for $y = -x$ only |

|            |  |   |   |
|------------|--|---|---|
| 15(a)(i)   | Correct translation  | 2 | <b>B1</b> for translation $\begin{pmatrix} 6 \\ k \end{pmatrix}$ or $\begin{pmatrix} k \\ -2 \end{pmatrix}$ |
| 15(a)(ii)  | Correct rotation   | 2 | <b>B1</b> for rotation $180^\circ$ but other centre   |
| 15(a)(iii) | Correct reflection   | 2 | <b>B1</b> for reflection in $y = -x$  |
| 15(b)(i)   | Enlargement<br>[factor] $\frac{1}{2}$ or $0.5$<br>[centre] $(0, 0)$ oe | 3 | <b>B1</b> for each  |
| 15(b)(ii)  | $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ oe  | 2 | <b>B1</b> for matrix of form $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ oe, $k \neq 0$ or $1$           |

|          |   |   |  |
|----------|---|---|--|
| 16 (i)   | $C$ at (3, 1) (3, 3) (4, 3)                         | 2 | <b>B1</b> for either vertical or horizontal correct<br>Or for two vertices correct and correct orientation                     |
| (ii)     | $y = x$ oe  | 1 |  |
| (iii)    | Translation $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ | 2 | <b>B1</b> for translation or $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$<br>Or<br><b>M1</b> for $D$ seen at (1, 3), (3, 3), (3, 4) |
| (iv) (a) | (2, 0) (4, 0) (4, -1)                               | 1 |  |
| (b)      | Rotation, $90^\circ$ clockwise, (0,0) oe            | 2 | <b>B1</b> for two correct from:<br>Rotation,<br>$90^\circ$ clockwise oe,<br>(0, 0) oe  |
| (c)      | $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$     | 1 |  |