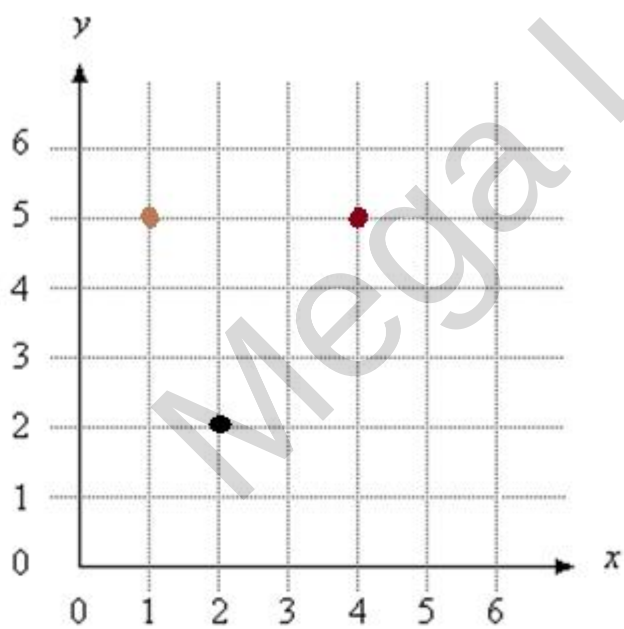
TRANSFORMATION

GENERAL KEY POINTS

-> When I talk about measuring distances from centre of rotation or enlargement, it's to count the number of boxes vertically and horizontally FROM the centre and till the point. All the distances obtained after the transformation has been applied, have to be plotted from the CENTRE AS WELL. Note that this point applies to rotation and enlargement. Reflection has its own concepts which are described under its heading later on.

-> If you're moving right horizontally from the centre, then it's a positive x distance and vice versa. If you're moving up, it's a positive y distance and vice versa.

Let's take an example of what I want to convey.



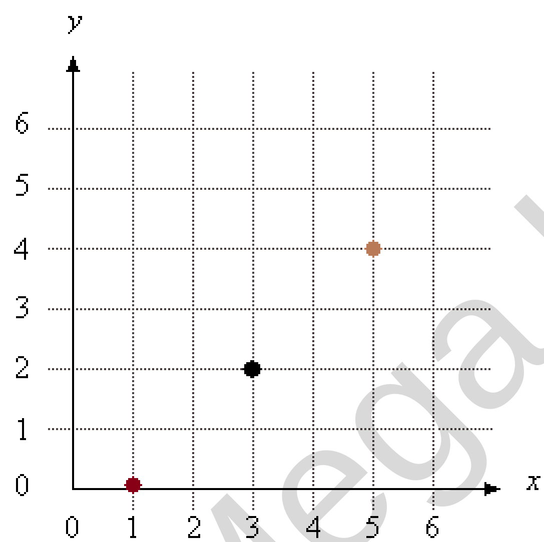
If you consider the black dot as the centre, then the x and y 'distances' of the red dot from the centre are 2 and 3 respectively. If you consider the orange dot, then the distances are -1 and 3 respectively.

ENLARGEMENT

For enlargement on a specific point, obtain the x and y distances from the centre as explained above and multiply 'EACH OF THEM' by the scale factor. You obtain new x and y distances. Plot them and you should reach your enlarged point.

WORKED EXAMPLE

The following example uses the black dot as the centre of enlargement, the orange dot as the point to be enlarged and the red dot as the enlarged point. The scale factor is -1 (I didn't have space to use a larger scale factor .-.)



The x and y distances of the orange dot from the black dot are 2 and 2. Multiplying them by the scale factor (-1) gives you -2 and -2. Mark these distances from the black dot and you arrive at the point (1,0). Therefore, the enlargement of (5,4) through the point (3,2) and by a scale factor of -1 is: (1,0). I hope this was easy to understand.

FINDING THE CENTRE OF ENLARGEMENT

To find the centre of enlargement, you need atleast two points and their enlarged points which have undergone the same transformation. Connect both of the points to their respective enlarged points. Where they intersect is the centre of enlargement.

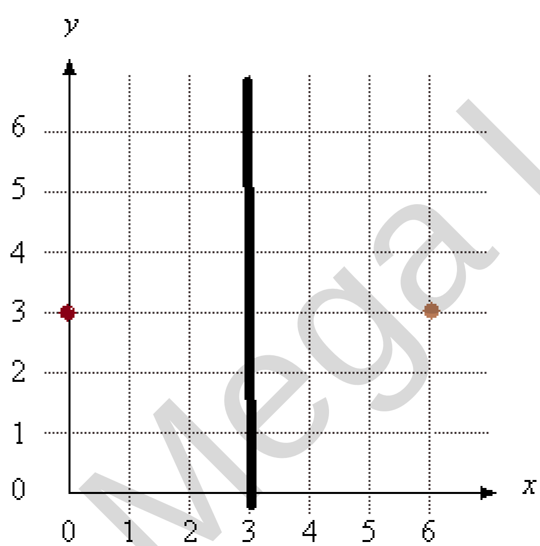
FINDING THE SCALE FACTOR

To find the scale factor, consider either x or y distances from the centre of enlargement of a specific point and its enlarged point. In the example above, the x distance of the orange dot is 2 and that of the red dot is -2. If you divide the distance of the enlarged point (red dot) by the distance of actual point (orange dot), you get the scale factor. In this case, -2 ÷ 2 = -1.

NOTE: A negative scale factor indicates that the image is formed on the opposite side of the object with respect to the centre of enlargement (can be seen in the example above). A positive scale factor indicates that both the image and object will be on the same side with respect to the centre of enlargement.

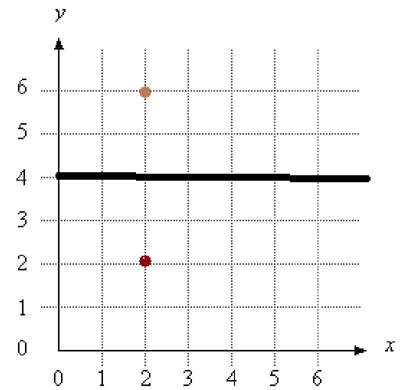
REFLECTION

REFLECTION IN VERTICAL LINES



Consider the following reflection in the line x = 3. To reflect points in vertical lines is pretty simple. Simply count the boxes perpendicularly to the line and the same number of number of boxes on the opposite side. In this case, if you see the orange dot, it's about 3 boxes away from the line of reflection and therefore, its reflected point (red dot) is also 3 boxes away.

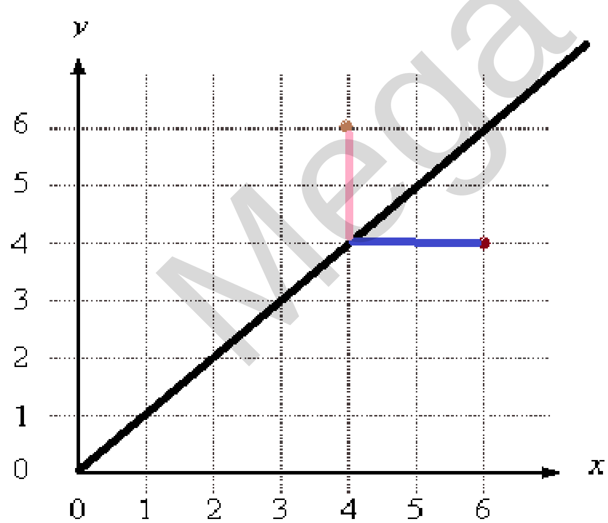
REFLECTION IN HORIZONTAL LINES



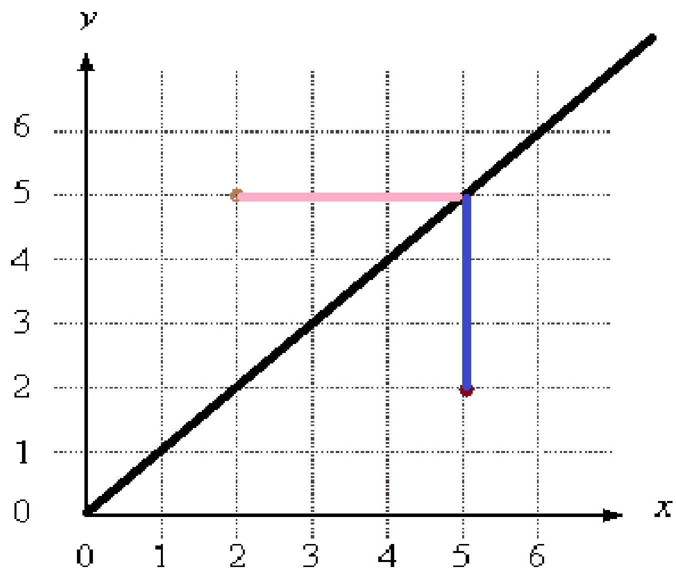
Pretty much the same idea as above. The image is inserted to give you an idea of how it's supposed to be done.

REFLECTION IN TITLED LINES

This is a bit tricky one and includes a switch in direction of counting boxes after striking the line of reflection. For instance, if you counted boxes from a point to the line horizontally, you would count the same number of boxes behind the line vertically to reach the point and vice versa.



Consider the red dot as the point to be reflected, count the number of boxes from it to the line. In this case, it's 2 boxes horizontally. Since we counted the boxes horizontally initially, we count them vertically after reaching the line to reach the orange dot which is our reflected point.



In this case, you count the number of boxes from the red dot to the line vertically at the start and once you reach the line, you switch your direction of counting and go horizontally on the other side of the line and mark the orange dot which is your reflected point.

NOTE: The actual point and reflected point lie on the opposite sides of the line.

FINDING THE LINE OF REFLECTION

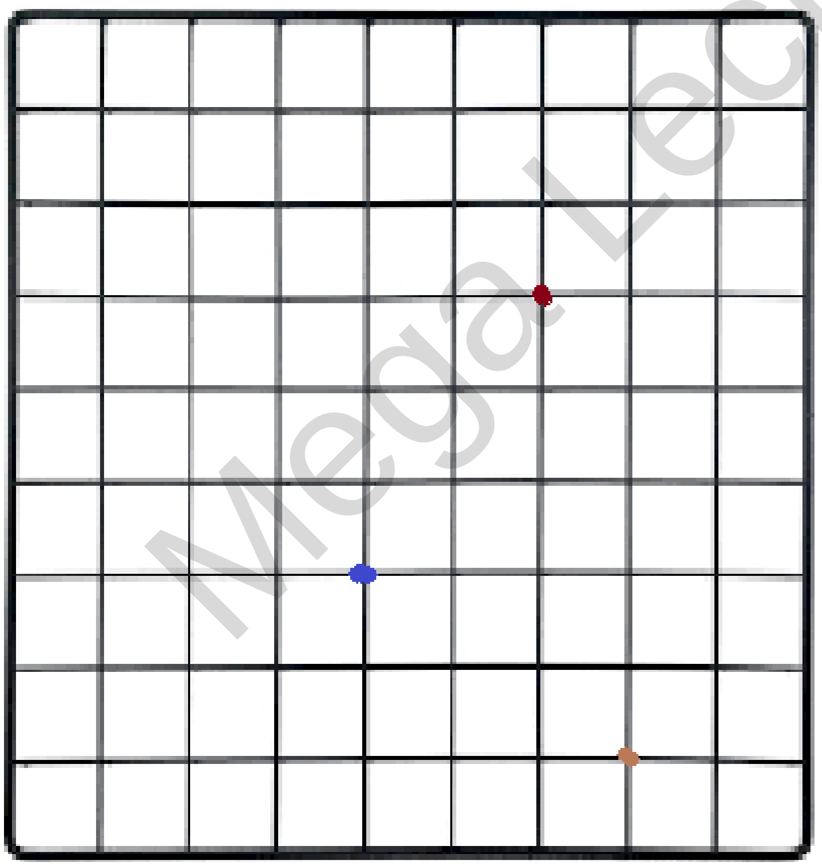
You need minimum 1 point and its reflected point. Join them and construct their perpendicular bisector as in loci. The perpendicular bisector is their line of reflection. If the reflected point and actual point have the same x coordinate, then that means that they have been reflected in a horizontal line. In this case, simply count the boxes between them and divide the total by 2 and draw a horizontal line at that very point. If the point and reflected point have the same y coordinate, they have been reflected in a vertical line and for that, count the total boxes, divide them by 2 and construct a vertical line at that point. For a condition which follows none of the above (tilted line), use the method explained at the beginning. You may require a compass to form an accurate line in this case.

ROTATION

NOTE: All distances are obtained FROM the centre of rotation and all distances after undergoing transformation, are also plotted FROM the centre of rotation. This point was mentioned at the beginning but this is just a reminder.

90 DEGREES CLOCKWISE

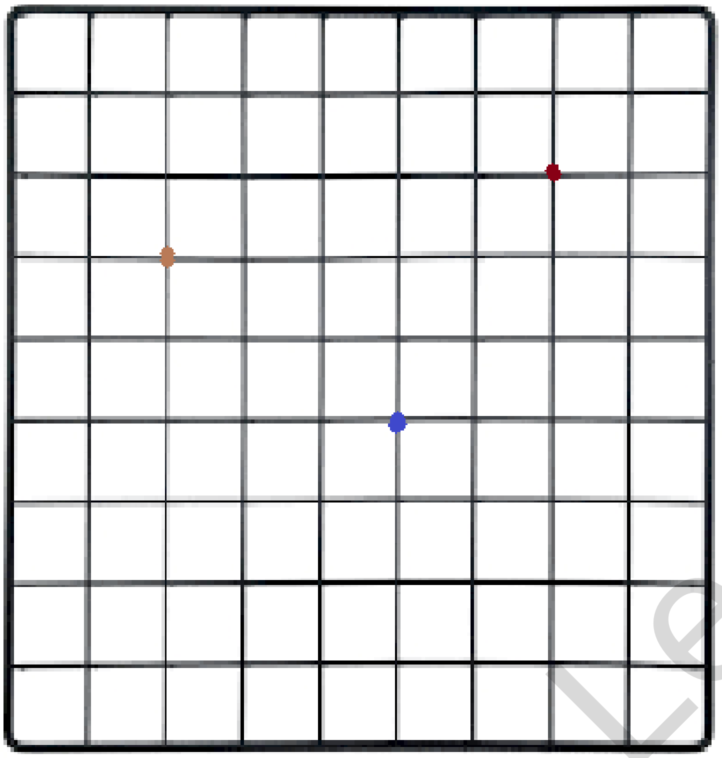
Mark the distances from the centre of rotation of a point in x and y directions. Flip the sign of the x distance and then swap both of the distances. For instance, you got x = 2 and y = 3. Flipping the sign of the x distance, we get x = -2 and then swapping both the distances we get, x = 3 and y = -2. Mark these final distances from the centre of rotation to arrive at the rotated point.



Here's an example. The red point is rotated about the blue point to obtain the rotated point (orange).

90 DEGREES ANTICLOCKWISE

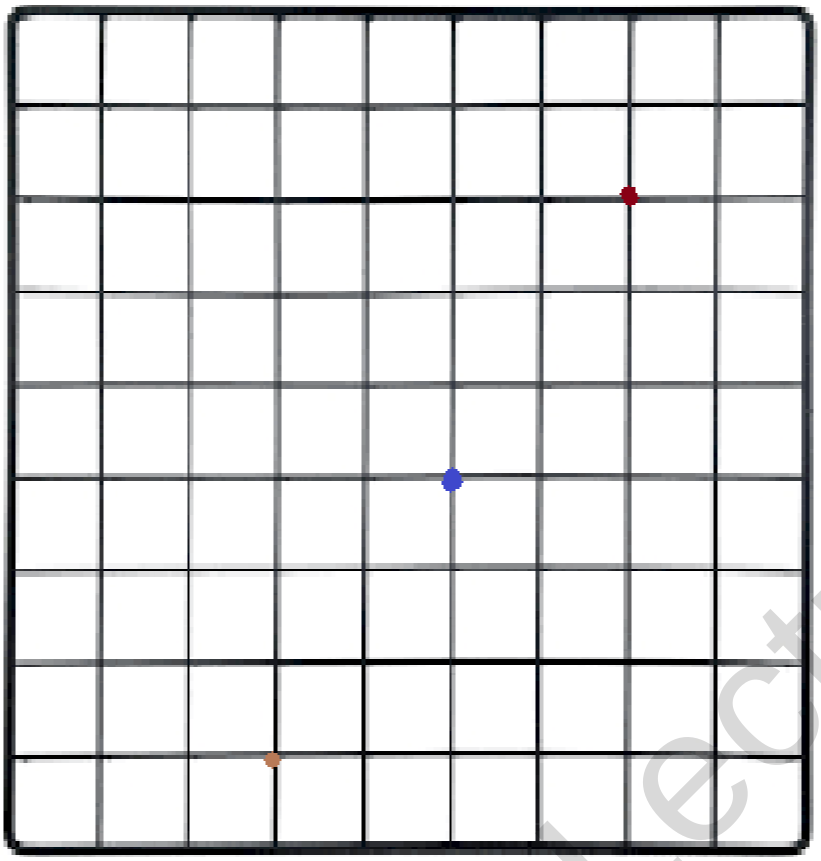
This is very similar to 90 degrees clockwise. Obtain the x and y distances from the centre of rotation. In this case, however, flip the sign of the y distance and then swap both the distances.



Here's an example. The red point is rotated about the blue point to obtain the rotated point (orange).

180 DEGREES

The thing about 180 degrees is that it doesn't matter whether you go clock or anti-clock. You arrive at the same point. Mark the distances in x and y directions, simply flip their signs and plot them again. You arrive at your 180 degrees rotated point.



Here's an example. The red point is rotated about the blue point to obtain the rotated point (orange).

CHEAT TIP 101

The examiner can trick you by asking you to rotate 270 degrees clock or anti clock but the thing to realize is that 270 degrees clock is actually 90 degrees anti-clock and vice versa. Keep that in mind!

FINDING THE CENTRE OF ROTATION

You need a minimum of 2 points with their rotated points. Join them to their respective rotated points. Construct the perpendicular bisector of each of the lines as in loci. The intersection of the 'perpendicular bisectors' is the centre of rotation.

FINDING THE ANGLE OF ROTATION

Connect the point to the centre of rotation and its rotated point to the centre of rotation. Measure the angle formed using a protractor.

MATRIX TRANSFORMATION AND TRANSLATION

Translation is basically just picking up an object and placing it somewhere else. It generally includes adding values to the x and y coordinates of an object to get the image. The values added are combined as in a column matrix known as the translation matrix. Essentially:

TRANSLATION MATRIX + OBJECT MATRIX = IMAGE MATRIX

The formula can be re-arranged to form desired equations to get a specific piece of information.

Other than translation, all other matrix transformations occur by pre-multiplying the object matrix by the transformation matrix. Essentially, for transformations other than translation:

TRANFORMATION MATRIX x OBJECT MATRIX = IMAGE MATRIX

There is no need to memorize any transformation matrix. All you need to use is the identity matrix which is: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The first column is the point (1,0) and the second column is point (0,1). Keeping the stuff I taught you above in mind, you can form matrices on your own, even for transformations independent of the origin!

You'd be wondering how to find the x and y distances when not given a graph paper (for rotation and enlargement). It's simple. Arrange the points (1,0) and (0,1) in a column matrix individually and that of the centre as well. Subtract the centre's coordinates from their coordinates to get the x and y distances.

For the (1,0) point with centre of rotation as origin: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ gives you the obvious x and y distances as 1 and 0 respectively. The same goes for the point: (0,1) whose x and y distances 0 and 1 respectively. Once the distances are found, apply the transformation, whose matrix is required, on the distances and then visualize how you would move from centre and in the x and y directions generated to obtain the POINT. For example, you are enlarging a figure by the origin and by a scale factor of 2. You need a matrix for that. The x and y distances of (1,0) from centre are 1 and 0. Multiplying that by scale factor gives 2 and 0. If you move 2 blocks in x direction (towards the right since it's positive) and 0 blocks upwards/downwards from the centre (which in this case is origin), you get the point (2,0). Do the same for the point (0,1) in the identity matrix. Once the new coordinates of (1,0) and (0,1) are found: insert them in the 2x2 matrix in which you write the transformed coordinates of (1,0) first and so on. This is the method you follow for a rotational or enlargement matrix. You may draw a small grid to help you out.

The only reflection matrices asked in the CAIE examinations are that of reflection in y-axis, x-axis, y = x and y = -x line. For the y = x line, the coordinates simply swap themselves. For instance, a point such as (2,3) would become (3,2) when reflected in y = x line. For the y = -x line, the coordinates not only swap themselves but also flip their signs. For instance, a coordinate (-2,3) would become (-3,2) after being reflected in the line y = -x. Based on this, you can easily form the matrices for the y = x and y = -x lines using the identity matrix where you write the transformed coordinates of (1,0) first.

For the reflection in y-axis, realize that the point (0,1) lies on the y-axis and therefore, no change occurs w.r.t it. As for the point (1,0), its distance from the y-axis is 1 block and therefore, the distance on the opposite side of the y-axis should be 1 block as well. Therefore, the point (1,0) when reflected in y-axis, is (-1,0). You can arrange them in a column matrix which represents the matrix for reflection in y-axis.

For the reflection in x-axis, same thing. The point (1,0) undergoes no transformation and the point (0,1), being 1 block away from the x-axis, becomes (0,-1).

CHARACTERISTICS OF TRANSFORMATIONS

TRANSFORMATION	SHAPE	ORIENTATION	SIZE
ENLARGEMENT	SAME	VARIES	VARIES

ROTATION	SAME	VARIES	SAME
REFLECTION	SAME	VARIES	SAME
TRANSLATION	SAME	SAME	SAME