1:J15/32/8

Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express f(x) in partial fractions. [5]

(ii) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

$A = 3, B = -1, C = -2, (ii)\frac{1}{2} + \frac{5}{12}x + \frac{41}{72}x^2$

2:J15/33/10

Let $f(x) = \frac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express f(x) in partial fractions. [5]

(ii) Show that $\int_1^2 f(x)\,dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

3:N14/31/9

(i) A =2, B = -1 C = 3 (ii) ln(2x - 1) – ln (x + 2) +(x - 1)/(x + 2)

Let $f(x) = \frac{x^2 - 8x + 9}{(1 - x)(2 - x)^2}$.

(i) Express f(x) in partial fractions. [5]

(ii) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [5]

4:J14/31/q9

$(i) A = 2, B = -1, C = 3\ (ii)\frac{9}{4} + \frac{5}{2}x + \frac{39}{16}x^2$

(i) Express $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

5:J14/33/q8

$(i) A = 1, B = \frac{3}{2}, C = \frac{-1}{2}\ (ii)\frac{4}{3} - \frac{8}{9}x + \frac{1}{27}x^2$

Let $f(x) = \frac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express f(x) in the form $\frac{A}{2 - x} + \frac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\int_{-1}^{1} f(x)\,dx = 3\ln 3$. [5]

6:N13/31/q7

Let $f(x) = \frac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$(i) A = -1, B = 3, C = -1 \; (ii) \frac{1}{6} + \frac{5}{4}x + \frac{17}{72}x^2$

7:N13/32/q7

Let $f(x) = \frac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$(i) A = -1, B = 3, C = -1 (ii) \frac{1}{6} + \frac{5}{4}x + \frac{17}{72}x^2$

8:N13/33/q8

(i) Express $\frac{7x^2 + 8}{(1+x)^2(2-3x)}$ in partial fractions. [5]

(ii) Hence expand $\frac{7x^2 + 8}{(1+x)^2(2-3x)}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [5]

$(i) A = -1, B = 3, C = 4 (ii) 4 - 2x + \frac{25}{2}x^2$

9:J13/31/q3

Express $\frac{7x^2 - 3x + 2}{x(x^2+1)}$ in partial fractions. [5]

$A = 2, B = 5, C = -3$

10:J13/32/q8

(i) Express $\frac{1}{x^2(2x+1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x+1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x+1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

$(i) A = 1, B = -2, C = 4 (ii) y = \frac{25}{36}e^{\frac{1}{2}}$

11:N12/33/q9

(i) Express $\frac{9 - 7x + 8x^2}{(3-x)(1+x^2)}$ in partial fractions. [5]

(i) Express $\frac{9-7x+8x^2}{(3-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{9-7x+8x^2}{(3-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

12:J12/31/q9

$(i)A = 6, B = -2, C = 1 (ii) 3 - \frac{4}{3}x - \frac{7}{9}x^2 + \frac{56}{27}x^3$

By first expressing $\frac{4x^2+5x+3}{2x^2+5x+2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2+5x+3}{2x^2+5x+2}\,dx = 8 - \ln 9.$$ [10]

13:J12/33/q8

$(i)A = 2, B = 1, C = -3$

Let $f(x) = \frac{4x^2-7x-1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_2^6 f(x)\,dx = 8 - \ln(\frac{49}{3})$. [5]

14:N11/31/q8

$(i)A = 2, B = -2, C = -1$

Let $f(x) = \frac{12+8x-x^2}{(2-x)(4+x^2)}$.

(i) Express $f(x)$ in the form $\frac{A}{2-x} + \frac{Bx+C}{4+x^2}$. [4]

(ii) Show that $\int_0^1 f(x)\,dx = \ln(\frac{25}{2})$. [5]

15:J11/32/Q8

$(i)A = 3, B = 4, C = 0$

(i) Express $\frac{5x-x^2}{(1+x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x-x^2}{(1+x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

16:N10/31/q8

$(i)a = -2, b = 1, c = 4 (ii) \frac{5}{2}x - 3x^2 + \frac{7}{4}x^3$

Let $f(x) = \frac{3x}{(1+x)(1+2x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

$(i) A = -1, B = 2, C = 1 (ii) 3x - 3x^2 - 3x^3$

17:J10/31/q8

(i) Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part (i), show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

(iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2}\,dx = \frac{7}{12} - \ln\frac{3}{2}$. [5]

18:J10/32/q10

$(i) A = 1, B = -1$

(i) Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x-1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x-1}.$$ [5]

(ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}\,dx = \frac{3}{2} + \frac{1}{2}\ln\left(\frac{16}{27}\right).$$ [5]

19:J10/33/q9

$(i) A = 1, B = 2, C = 1, D = -3$

(i) Express $\frac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

20:N09/31/q8

$(i) A = 1, B = 1, C = -2 (ii) 1 + \frac{9}{4}x + \frac{15}{4}x^2$

(i) Express $\frac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x+3}{(x+1)^2(3x+2)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

21:N09/32/q8

$(i) A = -3, B = 1, C = 2 (ii) \frac{3}{2} - \frac{11}{4}x + \frac{29}{8}x^2$

(i) Express $\frac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1+x}{(1-x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^2. [5]

22:J09/q8

$(i) A = \frac{2}{3}, B = \frac{2}{3}, C = \frac{1}{3} (ii) \frac{1}{2} + x + \frac{3}{4}x^2$

(i) Express $\frac{100}{x^2(10-x)}$ in partial fractions. [4]

(ii) Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{100}x^2(10-x),$$

obtaining an expression for t in terms of x. [6]

23:J08/q7

$(i) A = 1, B = 1, C = 10 (ii) \ln\left(\frac{9x}{10-x}\right) - \frac{10}{x} + 10$

Let $f(x) \equiv \frac{x^2+3x+3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^3 f(x)\,dx = 3 - \frac{1}{2}\ln 2$. [4]

24:N07/q9

(i) A = 1, B = ½, C = -3/2

(i) Express $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2-x+8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x, up to and including the term in x^2. [5]

25:N06/q8

$(i) A = 1, B = 2, C = -4 (ii) 1 - 2x + \frac{17}{2}x^2$

Let $f(x) = \frac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^2 f(x)\,dx = 2 + \ln\frac{5}{3}$. [5]

26:J06/q9

$(i) A = 2, B = -1, C = 3$

(i) Express $\frac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\frac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

$(i) A = 2, B = 2, C = 4 (ii) 5 + \frac{5}{2}x - \frac{15}{4}x^2 - \frac{15}{8}x^3$

27:N05/q9

(i) Express $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

28:N04/q8

$(i) A = 2, B = 1, C = -1 (ii) \frac{1}{2}x + \frac{5}{4}x^2 - \frac{9}{8}x^3$

An appropriate form for expressing $\dfrac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\dfrac{4x}{(x+4)(x^2+3)}$, [1]

(ii) $\dfrac{2x+1}{(x-2)(x+2)^2}$. [2]

(b) Show that $\displaystyle\int_3^4 \frac{3x}{(x+1)(x-2)}\,dx = \ln 5$. [6]

29:J04/q9

Let $f(x) = \dfrac{x^2+7x-6}{(x-1)(x-2)(x+1)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = -3 + 2x - \tfrac{3}{2}x^2 + \tfrac{11}{4}x^3.$$ [5]

30:N03/q8

$(i) A = -1, B = 4, C = -2$

Let $f(x) = \dfrac{x^3-x-2}{(x-1)(x^2+1)}$.

(i) Express $f(x)$ in the form

$$A + \frac{B}{x-1} + \frac{Cx+D}{x^2+1},$$

where A, B, C and D are constants. [5]

(ii) Hence show that $\displaystyle\int_2^3 f(x)\,dx = 1$. [4]

Let $f(x) = \dfrac{9x^2 + 4}{(2x + 1)(x - 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that, when x is sufficiently small for x^3 and higher powers to be neglected,

$$f(x) = 1 - x + 5x^2.$$ [4]

32:N02/q6

$(i) A = 1, B = 4, C = 8$

Let $f(x) = \dfrac{6 + 7x}{(2 - x)(1 + x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = 3 + 5x - \tfrac{1}{2}x^2 - \tfrac{15}{4}x^3.$$ [5]

33:J02/q6

$(i) A = 4, B = 4, C = 1$

Let $f(x) = \dfrac{4x}{(3x + 1)(x + 1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = 1 - \ln 2$. [5]

$(i) A = -3, B = 1, C = 2$