Additional Mathematics SA2 Overall Revision Notes

Chapters 1 – 2 (   )

Simultaneous Equations, Indices, Surds, Logarithms

Chapter 1: Simultaneous Equations

There are 3 methods in solving simultaneous linear equations:

1.) Substitution Method
2.) Elimination Method
3.) Graphical Method

There are several steps to follow:

1.) Express one unknown in terms of another unknown (avoid fractional expressions)
2.) Substitute this newly – formed equation into the non-linear equation
3.) Solve for the unknown
4.) Use the linear equation to find the other unknown.

Chapter 2.1: Surds

$$\sqrt{m} \times \sqrt{n} = \sqrt{mn}$$

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{\frac{m}{n}}$$

$$a\sqrt{m} + b\sqrt{m} = a + b\sqrt{m}$$

$$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

$$a + b\sqrt{k} = c + d\sqrt{k}$$

$a = c$ and $b = d$.

Rationalising Denominator:
Multiply the square root to both numerator and denominator.

Chapter 2.2: Indices

$$a^m \times a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^m \times b^m = (ab)^m$$

$$a^m \div a^n = a^{m-n}$$

$$a^m \div b^m = (\frac{a}{b})^m$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$x(a^{-n}) = \frac{x}{a^n}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$a^x = a^n$$

$$\therefore x = n$$

When a > 1

Chapter 2.3: Logarithms

No.	Rules of Logarithms (base a)	Rules of Common Logarithms	Rules of Natural Logarithms
1.	$x = \log_a y \Leftrightarrow y = a^x$ $y > 0$ $(a > 0, a \neq 1)$	$x = \lg y \Leftrightarrow y = 10^x$ $y > 0$ (base 10) $\lg y = \log_{10} y$	$x = \ln y \Leftrightarrow y = e^x$ $y > 0$ (base e) $\ln y = \log_e y$ $e = 2.71828...$
2.	$\log_a a = 1$ $\log_a 1 = 0$ $a^{\log_a x} = x$	$\lg 10 = 1$ $\lg 1 = 0$ $10^{\lg x} = x$	$\ln e = 1$ $\ln 1 = 0$ $e^{\ln x} = x$
3.	$\log_a xy = \log_a x + \log_a y$	$\lg xy = \lg x + \lg y$	$\ln xy = \ln x + \ln y$
4.	$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\lg \left(\frac{x}{y}\right) = \lg x - \lg y$	$\ln \left(\frac{x}{y}\right) = \ln x - \ln y$
5.	$\log_a x^n = n \log_a x$	$\lg x^n = n \lg x$	$\ln x^n = n \ln x$
	Antilogarithms: a^x	10^x	e^x
6.	$\log_a p = \log_a q \Leftrightarrow p = q$	$\lg p = \lg q \Leftrightarrow p = q$	$\ln p = \ln q \Leftrightarrow p = q$
7.	Change of base $\log_a b = \frac{\log_c b}{\log_c a}$	$\log_a b = \frac{\lg b}{\lg a}$	$\log_a b = \frac{\ln b}{\ln a}$
8.	Reciprocal $\log_a b = \frac{1}{\log_b a}$	$\log_x 10 = \frac{1}{\log_{10} x} = \frac{1}{\lg x}$	$\log_x e = \frac{1}{\log_e x} = \frac{1}{\ln x}$

Additional Mathematics SA2 Overall Revision Notes

Chapters 3 - 4

Quadratic Functions and Inequalities

Sum and Product of Roots

In $ax^2 + bx + c$

Sum of roots $\alpha + \beta = -\frac{b}{a}$

Product of roots $\alpha\beta = \frac{c}{a}$

We can use the sum and product of roots to write an equation.

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

Intersection Terms

Crosses / Cuts	2 points of intersection, 2 real/distinct roots/ discriminant more than 0.
Touches / tangent	1 point of intersection, 2 real/equal roots/ discriminant = 0.
Does not intersect / meet	0 points of intersection, no real roots, discriminant < 0.
Meet	Discriminant more than or equal to 0.

Quadratic Inequality

$(x-a)(x-b) > 0, x < a \text{ or } x > b$

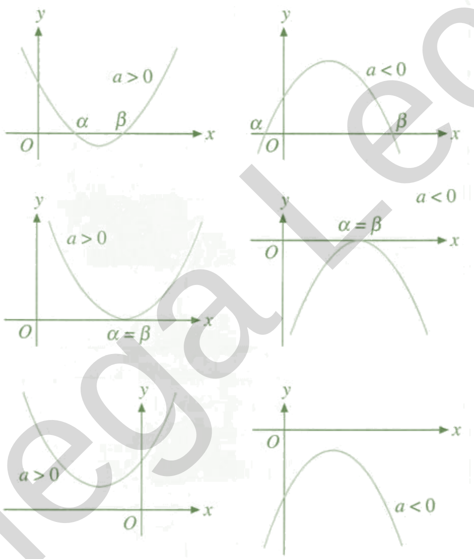
$(x-a)(x-b) \leq 0, a \leq x \leq b$

Discriminant and Nature of Roots

(a) $b^2 - 4ac > 0$
Two distinct real roots
$\Rightarrow$ two x-intercepts

(b) $b^2 - 4ac = 0$
Equal real roots
$\Rightarrow$ only one x-intercept and the x-axis is a tangent to the parabola

(c) $b^2 - 4ac < 0$
No real roots
$\Rightarrow$ no x-intercept and $y = ax^2 + bx + c$ is either always positive or always negative



Chapter 8: Linear Law

The graph of a linear equation Y = mX + c is a straight line with gradient m and y intercept *c*.
There are 2 parts to solving linear law questions: Draw a straight line graph to determine gradient and y-intercept, and to find the equation of the straight line.

Key Steps:

1.) Force the equation into the form of Y = mX + c.
2.) Take some experimental values of *x* and *y* and compute the corresponding values of *X* and *Y*.
3.) Use these computed values to plot the points on a graph with *X* and *Y* axis.
4.) Draw a line passing through the plotted points. Always have more space at the lower end of graph for the line to cut the Y axis for Y-intercept.
5.) Obtain the Gradient and the Y-intercept.

Note: In Y = mX + c

(a): Y must not have any coefficient,
(b): mX is part constant and part variable.
(c): c must not contain any variable X and Y.

Polynomials/Partial Fractions

Polynomial → An expression that is a sum of terms in the form ax^n where n is non-negative and a is constant.

To find unknown constants, either equate coefficients of like powers of x or substitute values of x.

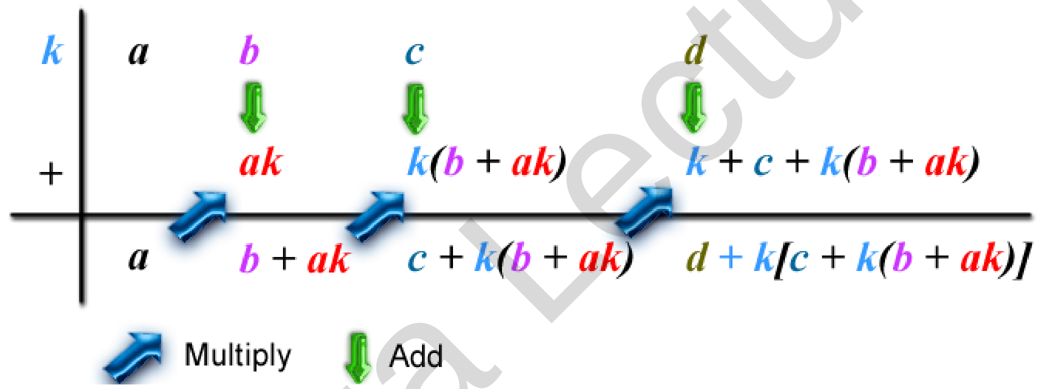
Remainder Theorem

If a polynomial f(x) is divided by a linear divisor ($x - a$), the remainder is f(a).

Factor Theorem

If ($x - a$) is a factor of the polynomial f(x), f(a) = 0.

Synthetic Method: $f(x)$ is divided by $(x - k)$



Partial Fractions

g(x) has	Corresponding Partial Fraction(s)
linear factor $ax + b$	$\frac{A}{ax+b}$
repeated linear factor $(ax + b)^2$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
quadratic factor $x^2 + c^2$ (which cannot be factorised)	$\frac{Ax+B}{x^2+c^2}$

Basically, a linear factor that cannot be factorised is to be remained in the same form. A repeated linear factor like $(ax+b)^2$ is to be split into 2: $\frac{A}{(ax+b)} + \frac{B}{(ax+b)^2}$.

Chapter 5: The Modulus Functions

For a real number x, |x| represents the modulus / absolute value of x. It is always non-negative.

To draw a modulus graph of the function, first draw the function then **reflect** the part of the function which is below the x axis **upwards**.

Formulas:

$$|x| = k \Rightarrow x = k \text{ or } x = -k$$

$$|f(x)| = \pm g(x),\ g(x) \geq 0$$

$$|f(x)| = |g(x)|,\ f(x) = \pm g(x)$$

$$|ab| = |a||b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

Chapter 6: Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 \ldots + b^n$$

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n-1}x^{n-1} + x^n$$

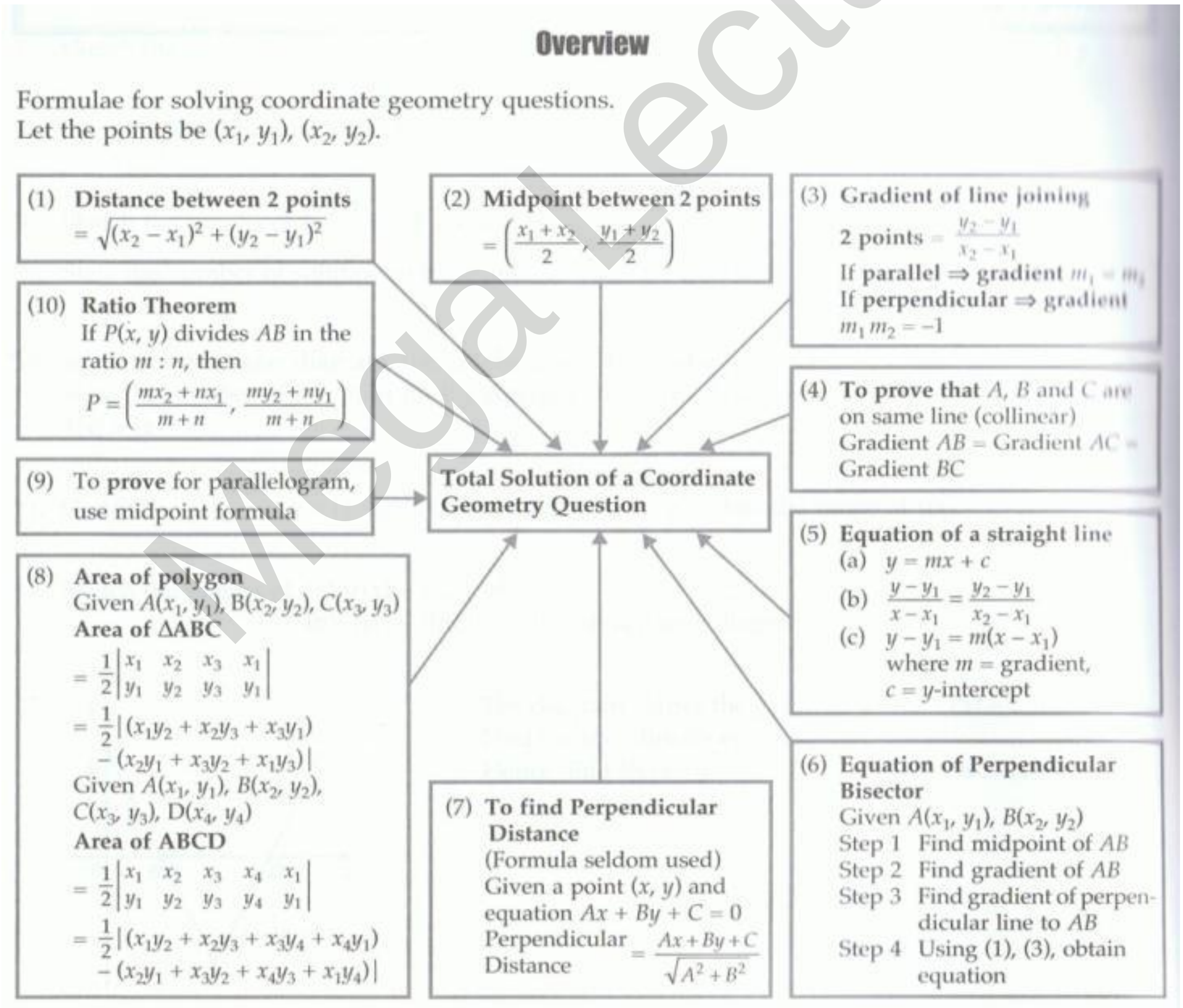
$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$$

Properties:

1.) Have n+1 terms
2.) Sum of powers of *a* and *b* = n.

r+1th term: $T_{r+1} = \binom{n}{r}a^{n-r}b^r$ or $T_{r+1} = \binom{n}{r}b^r$

Chapter 7: Coordinate Geometry



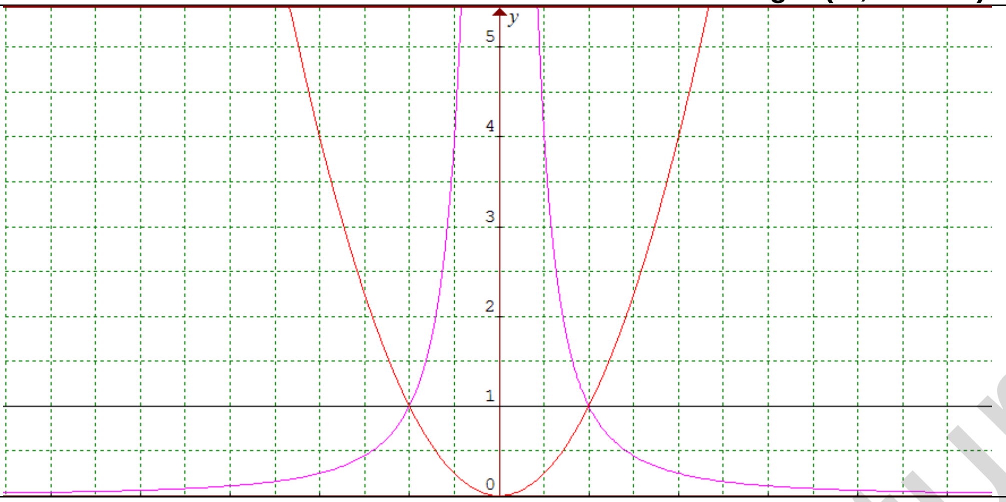
Additional Mathematics

Chapter 9

Curves and Circles (Summary)

Chapter 9.1: Graphs of $y = ax^n$

When n is an even integer (-2, 0 and 2)



Legend:

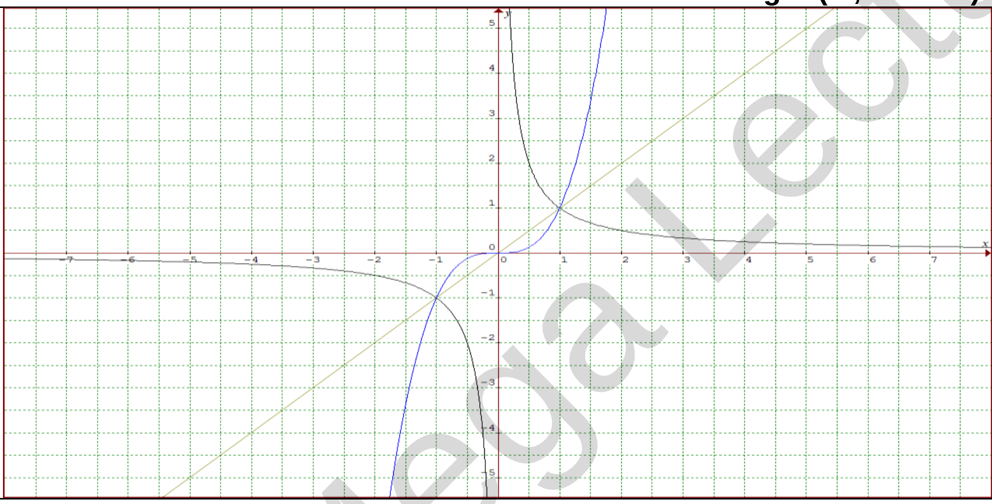
Red: $y = x^2$

Black: $y = x^0 = 1$

Pink: $y = x^{-2} = \frac{1}{x^2}$

1. Each curve is **above or on the x axis.**
2. Each curve is **symmetrical** about the x axis.
3. For the pink graph, it does not cut x or y axis.

When n is an odd integer (-1, 1 and 3)



Legend:

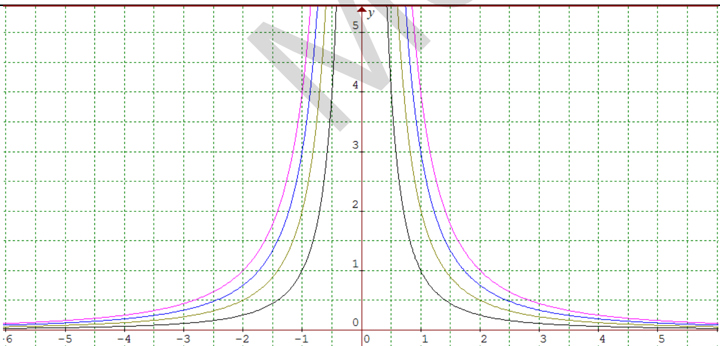
Blue: $y = x^3$

Brown: $y = x^1 = x$

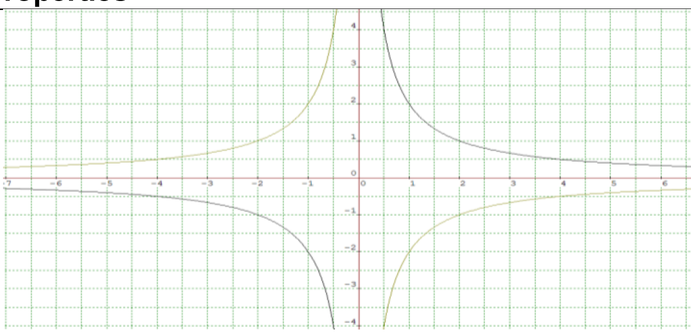
Black: $y = x^{-1} = \frac{1}{x}$

4. Each curve is **symmetrical** about the origin.

General Properties



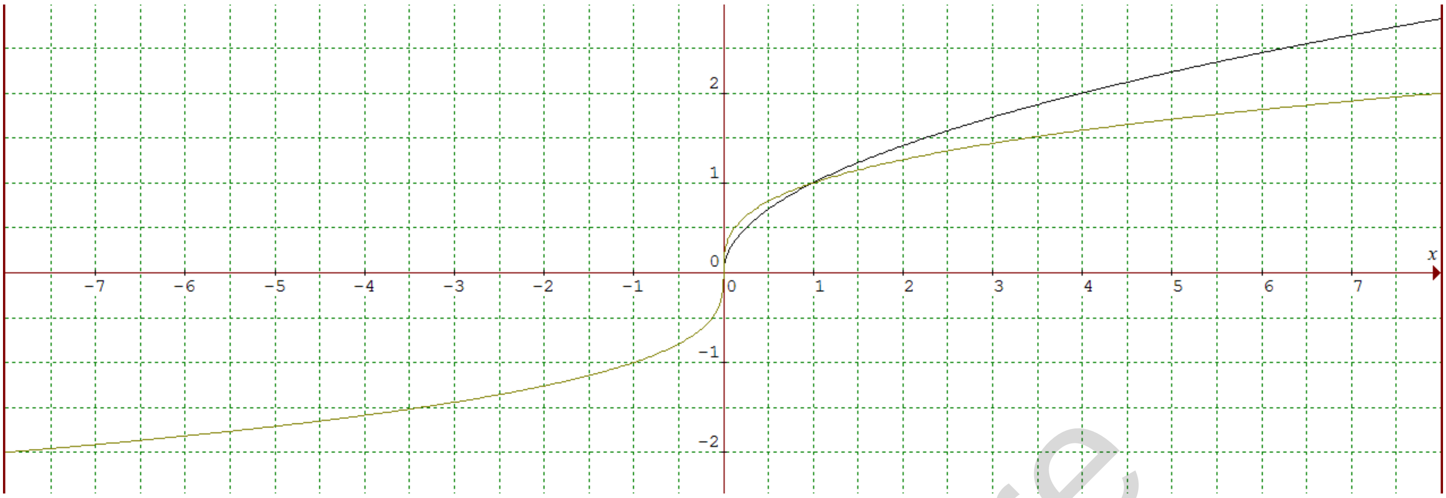
When a is constant, the graphs of $y = ax^n$ are similar except that they differ in the steepness as seen in the graphs of $y = x^{-2}$.



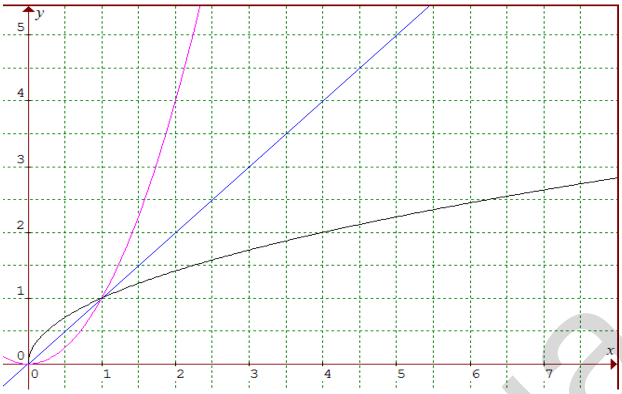
If a < 0, then the graph of $y = ax^n$ is a reflection of the graph of $y = |a|x^n$ in the x axis.

2 Graphs of $y = ax^n$ where n is a simple rational number

2. For $y = \sqrt{x}$ or $y = x^{\frac{1}{2}}$, x will be more or equal to 0 (x cannot be less than 0). y is also more than 0 as square root is taken to be positive.



Legend: Black: $y = \sqrt{x}$. Brown: $y = \sqrt[3]{x}$.

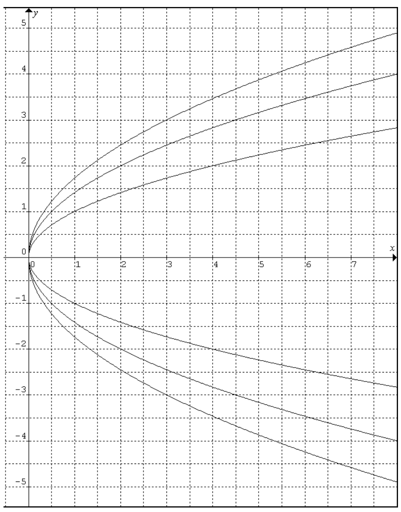


2. Comparing concavity of curves.

When $y = \sqrt{x}$, graph concaves downwards.
When $y = x$, graph is straight and constant.
When $y = x^2$, graph concaves upwards.

3 Graph of $y^2 = kx$

1. The graph of $y^2 = x$ is actually a 90 degree clockwise rotation of the graph of $y = x^2$ about the origin O.
2. In general, the graphs of $y^2 = kx$ have the same properties as that of $y^2 = x$ except that they differ in the steepness.
3. Each graph passes through (0, 0) and is symmetrical about the *x* axis.



4 Equations of Circles

Equation	$(x-a)^2 + (y-b)^2 = r^2$	$x^2 + y^2 + 2gx + 2fy + c = 0$
Center of circle	*(a, b)*	*(-g, -f)*
Radius	*r*	$\sqrt{g^2 + f^2 - c}$

5 Linear Law (Revision)

Always make an equation to Y = mX + c. (where m and c must be constant!)

Additional Mathematics

Chapter 11 and 12

Trigonometry Functions, Simple Trigonometric Identities/Equations

Chapter 11.1: Angle in Radian Measure

$180^\circ = \pi \text{ rad}$

$1^\circ = \dfrac{\pi}{180} \text{ rad}$

$1 \text{ rad} = \dfrac{180}{\pi} \approx 57.3^\circ$

Chapter 11.3: Trigonometric Ratios of Complimentary Angles

$\sin(90^\circ - \theta) = \cos\theta$	$\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$	$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$
$\tan(90^\circ - \theta) = \dfrac{1}{\tan\theta}$	$\tan\left(\dfrac{\pi}{2} - \theta\right) = \dfrac{1}{\tan\theta}$

Chapter 11.2: Trigonometric Ratios for Acute Angles

Just remember that the surd form of these numbers:

$\dfrac{\sqrt{3}}{3} \approx 0.577$

$\dfrac{\sqrt{2}}{2} \approx 0.707$

$\dfrac{\sqrt{3}}{2} \approx 0.806$

Chapter 11.4: Trigonometric Ratios of General Angles

The acute angle formed when a line rotates about the origin is called the **basic angle**, denoted by α. Always make the basic angle positive.

1^{st} Quadrant	2^{nd} Quadrant	3^{rd} Quadrant	4^{th} Quadrant
$\alpha = \theta$	$\alpha = 180^\circ - \theta$ $\alpha = \pi - \theta$	$\alpha = 180^\circ + \theta$ $\alpha = \pi + \theta$	$\alpha = 360^\circ - \theta$ $\alpha = 2\pi - \theta$

Chapter 11.5: Trigonometric Ratios of their General Angles and their Signs

In the 1^{st} quadrant, all 3 are positive.
In the 2^{nd} quadrant, only tangent is positive.
In the 3^{rd} quadrant, only sine is positive.
In the 4^{th} quadrant, only cosine is positive.
If still turning anticlockwise after 4^{th} quad, add 360° or 2π.

S	A
T	C

Chapter 11.6: Trigonometric Ratios of Negative Angles

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

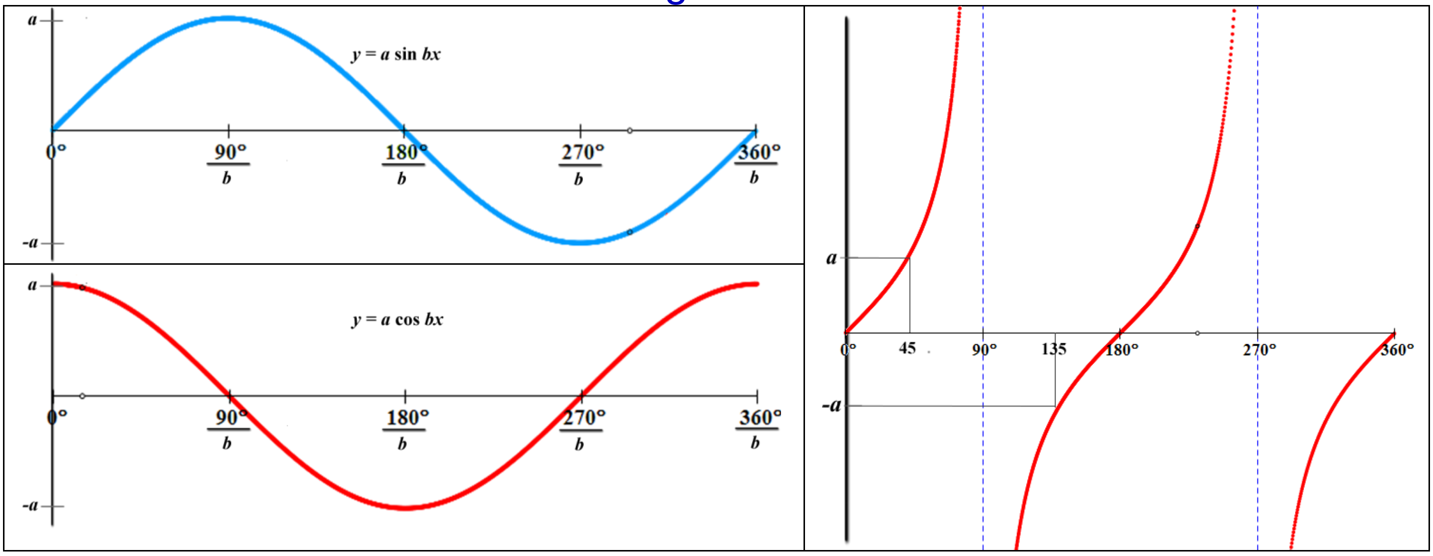
Chapter 11.7: Solving Basic Trigonometric Equations

1.) By considering the sign of *k*, identify the possible quadrants where theta will lie.
2.) Find the basic angle alpha, the acute angle from e.g.: $\sin\theta = |k|$
3.) Find all the possible values of theta in the given interval.

Chapter 11.8: Graphs of the sine, cosine and tangent functions

In general, the curves $y = a\sin bx + c$ and $y = a\cos bx + c$ have axis $y = c$, amplitude a and period $\dfrac{360^\circ \text{ or } 2\pi}{b}$

Graphs are shown on the next page.



Chapter 12.1: Summary of Identities

Simple Trigonometric Identities and Equations

Basic Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocals of 3 trigo functions:

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

The "Squared Ratios"

$$\sin^2\theta + \cos^2\theta = 1$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

In proving a trigonometric identity, **always start from the more complicated side (with the secant, cosecant and cotangent)**. The rest of the proving is all mechanical in nature!