ADD MATHS WORKSHEET # 1

1. Find the minimum value of $(x-2)^2 - 2$ and the corresponding value of x. Sketch the graph of $y = |(x-2)^2 - 2|$ for $0 \le x \le 4$. (N99/P1/17b)

2. Draw the graph of $y = 3 + |x^2 - 5x + 4|$, for $0 \le x \le 5$, using a scale of 2 cm to 1 unit along each axis. Use your graph to find the set of values of x for which $y \le 5$. (N01/P1/13b)

3. The function f is defined by $f: x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant 8$.

   (i) By first considering the stationary value of the function $x \mapsto x^2 - 8x + 7$, show that the graph of $y = f(x)$ has a stationary point at $x = 4$ and determine the nature of this stationary point.

   (ii) Sketch the graph of $y = f(x)$.

   (iii) Find the range of f.

The function g is defined by $g: x \mapsto |x^2 - 8x + 7|$ for the domain $3 \leqslant x \leqslant k$.

(iv) Determine the largest value of k for which g^{-1} exists. (N2004/P2/10)

4. The function f is defined for the domain $-3 \leqslant x \leqslant 3$ by $f(x) = 9\left(x - \frac{1}{3}\right)^2 - 11$.

   (i) Find the range of f.

   (ii) State the coordinates and nature of the turning point of

   (a) the curve $y = f(x)$,

   (b) the curve $y = |f(x)|$. (N2006/P1/7)

1. Find the range of values of x for which $3x^2 - 5x + 4 > 3 - x^2$. (N97/P1/4)

2. Find the range of values of x for which $x(10 - x) \geq 24$. (N98/P1/3a)

3. Find the range of values of x for which $3(x + 1)^2 < 16x$. (N99/P1/2)

4. Find the range of values of x for which $(2x + 1)(4 - x) > 4$. (N2000/P1/3b)

5. Given that $f(x) \equiv 2x^2 - 5x - 7$,

   (i) find the value of a, of b and of c for which $f(x) \equiv a(x - b)^2 - c$,

   (ii) state the minimum value of $f(x)$,

   (iii) sketch the graph of $y = |f(x)|$ for $-2 \leq x \leq 4{\cdot}5$, indicating on your graph the coordinates of the stationary point and of the points where the graph meets the coordinate axes,

   (iv) calculate the values of x for which $|f(x)| = 7$, giving your answers to 2 decimal places where appropriate. (N2000/P1/17)

6. Find the range of values of x for which $x(2x + 5) > 12$. (N01/P1/7a)

7. Find the x-coordinate of the point on the line $y = 5 - 2x$ where xy is a maximum. (N01/P1/15a)

6. Find the values of m for which the line $y = mx - 9$ is a tangent to the curve $x^2 = 4y$. (N2002/P1/2)

7. Find the values of k for which the line $x + 3y = k$ and the curve $y^2 = 2x + 3$ do not intersect. (N2003/P1/1)

8. Find the values of k for which the line $y = x + 2$ meets the curve $y^2 + (x + k)^2 = 2$. (N2004/P2/4)

9. Find the value of m for which the line $y = mx - 3$ is a tangent to the curve $y = x + \frac{1}{x}$ and find the x-coordinate of the point at which this tangent touches the curve. (N2006/P2/7a)

10. The equation of a straight line is $y = 5 + kx$, where k is a constant. Find the values of k for which this straight line is a tangent to the curve $y^2 = 4y + x + 1$. (SP08/P1/4)

8. **EITHER**

Functions f and g are defined for $x \in \mathbb{R}$ by

$f : x \mapsto 3x - 2, x \neq \frac{4}{3}$,

$g : x \mapsto \frac{4}{2-x}, x \neq 2$.

(i) Solve the equation $gf(x) = 2$.

(ii) Determine the number of real roots of the equation $f(x) = g(x)$.

(iii) Express f^{-1} and g^{-1} in terms of x.

(iv) Sketch, on a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, stating the coordinates of the point of intersection of the two graphs.

OR

(i) Find the value of a and of b for which $1 - x^2 + 6x$ can be expressed in the form $a - (x + b)^2$.

A function f is defined by $f : x \mapsto 1 - x^2 + 6x$ for the domain $x \geqslant 4$.

(ii) Explain why f has an inverse.

(iii) Find an expression for f^{-1} in terms of x.

A function g is defined by $g : x \mapsto 1 - x^2 + 6x$ for the domain $2 \leqslant x \leqslant 7$.

(iv) Find the range of g.

(v) Sketch the graph of $y = |g(x)|$ for $2 \leqslant x \leqslant 7$. (N2003/P2/12)

10. The functions f and g are defined for $x \in \mathbb{R}$ by $f : x \mapsto x^3$,

$g : x \mapsto x + 2$.

Express each of the following as a composite function, using only f, g, f^{-1} and/or g^{-1} :

(i) $x \mapsto x^3 + 2$,

(ii) $x \mapsto x^3 - 2$,

(iii) $x \mapsto (x + 2)^{\frac{1}{3}}$. (N2006/P2/1)