

### P3 (variant1 and 3)

#### Q1.

- 10 The lines  $l$  and  $m$  have vector equations

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

respectively.

- (i) Show that  $l$  and  $m$  intersect. [4]
- (ii) Calculate the acute angle between the lines. [3]
- (iii) Find the equation of the plane containing  $l$  and  $m$ , giving your answer in the form  $ax + by + cz = d$ . [5]

#### Q2.

- 10 The straight line  $l$  has equation  $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ . The plane  $p$  has equation  $3x - y + 2z = 9$ . The line  $l$  intersects the plane  $p$  at the point  $A$ .

- (i) Find the position vector of  $A$ . [3]
- (ii) Find the acute angle between  $l$  and  $p$ . [4]
- (iii) Find an equation for the plane which contains  $l$  and is perpendicular to  $p$ , giving your answer in the form  $ax + by + cz = d$ . [5]

#### Q3.

- 3 Points  $A$  and  $B$  have coordinates  $(-1, 2, 5)$  and  $(2, -2, 11)$  respectively. The plane  $p$  passes through  $B$  and is perpendicular to  $AB$ .

- (i) Find an equation of  $p$ , giving your answer in the form  $ax + by + cz = d$ . [3]
- (ii) Find the acute angle between  $p$  and the  $y$ -axis. [4]

#### Q4.

- 10 With respect to the origin  $O$ , the lines  $l$  and  $m$  have vector equations  $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$  and  $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$  respectively.
- (i) Prove that  $l$  and  $m$  do not intersect. [4]
  - (ii) Calculate the acute angle between the directions of  $l$  and  $m$ . [3]
  - (iii) Find the equation of the plane which is parallel to  $l$  and contains  $m$ , giving your answer in the form  $ax + by + cz = d$ . [5]

### Q5.

- 8 The point  $P$  has coordinates  $(-1, 4, 11)$  and the line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ .
- (i) Find the perpendicular distance from  $P$  to  $l$ . [4]
  - (ii) Find the equation of the plane which contains  $P$  and  $l$ , giving your answer in the form  $ax + by + cz = d$ , where  $a, b, c$  and  $d$  are integers. [5]

### Q6.

- 9 The lines  $l$  and  $m$  have equations  $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$  and  $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$  respectively, where  $a$  and  $b$  are constants.
- (i) Given that  $l$  and  $m$  intersect, show that
$$2a - b = 4. \quad [4]$$
  - (ii) Given also that  $l$  and  $m$  are perpendicular, find the values of  $a$  and  $b$ . [4]
  - (iii) When  $a$  and  $b$  have these values, find the position vector of the point of intersection of  $l$  and  $m$ . [2]

### Q7.

- 6 The points  $P$  and  $Q$  have position vectors, relative to the origin  $O$ , given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of  $PQ$  is the point  $A$ . The plane  $\Pi$  is perpendicular to the line  $PQ$  and passes through  $A$ .

- (i) Find the equation of  $\Pi$ , giving your answer in the form  $ax + by + cz = d$ . [4]
- (ii) The straight line through  $P$  parallel to the  $x$ -axis meets  $\Pi$  at the point  $B$ . Find the distance  $AB$ , correct to 3 significant figures. [5]

**Q8.**

- 10 The line  $l$  has equation  $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ , where  $a$  is a constant. The plane  $p$  has equation  $x + 2y + 2z = 6$ . Find the value or values of  $a$  in each of the following cases.
- (i) The line  $l$  is parallel to the plane  $p$ . [2]
  - (ii) The line  $l$  intersects the line passing through the points with position vectors  $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$  and  $\mathbf{i} + \mathbf{j} - \mathbf{k}$ . [4]
  - (iii) The acute angle between the line  $l$  and the plane  $p$  is  $\tan^{-1} 2$ . [5]

**Q9.**

- 6 With respect to the origin  $O$ , the points  $A$ ,  $B$  and  $C$  have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

The mid-point of  $AB$  is  $M$ . The point  $N$  lies on  $AC$  between  $A$  and  $C$  and is such that  $AN = 2NC$ .

- (i) Find a vector equation of the line  $MN$ . [4]
- (ii) It is given that  $MN$  intersects  $BC$  at the point  $P$ . Find the position vector of  $P$ . [4]

**Q10.**

- 7 With respect to the origin  $O$ , the points  $A$  and  $B$  have position vectors given by  $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  and  $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$ . The point  $P$  lies on the line  $AB$  and  $OP$  is perpendicular to  $AB$ .

- (i) Find a vector equation for the line  $AB$ . [1]
- (ii) Find the position vector of  $P$ . [4]
- (iii) Find the equation of the plane which contains  $AB$  and which is perpendicular to the plane  $OAB$ , giving your answer in the form  $ax + by + cz = d$ . [4]

**Q11.**

- 6 The straight line  $l$  passes through the points with coordinates  $(-5, 3, 6)$  and  $(5, 8, 1)$ . The plane  $p$  has equation  $2x - y + 4z = 9$ .

- (i) Find the coordinates of the point of intersection of  $l$  and  $p$ . [4]
- (ii) Find the acute angle between  $l$  and  $p$ . [4]

**Q12.**

- 7 With respect to the origin  $O$ , the position vectors of two points  $A$  and  $B$  are given by  $\vec{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$  and  $\vec{OB} = 3\mathbf{i} + 4\mathbf{j}$ . The point  $P$  lies on the line through  $A$  and  $B$ , and  $\vec{AP} = \lambda \vec{AB}$ .
- (i) Show that  $\vec{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$ . [2]
- (ii) By equating expressions for  $\cos AOP$  and  $\cos BOP$  in terms of  $\lambda$ , find the value of  $\lambda$  for which  $OP$  bisects the angle  $AOB$ . [5]
- (iii) When  $\lambda$  has this value, verify that  $AP : PB = OA : OB$ . [1]

### Q13.

- 9 The line  $l$  has equation  $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ , where  $a$  is a constant. The plane  $p$  has equation  $2x - 2y + z = 10$ .
- (i) Given that  $l$  does not lie in  $p$ , show that  $l$  is parallel to  $p$ . [2]
- (ii) Find the value of  $a$  for which  $l$  lies in  $p$ . [2]
- (iii) It is now given that the distance between  $l$  and  $p$  is 6. Find the possible values of  $a$ . [5]

### Q14.

- 10 With respect to the origin  $O$ , the points  $A$ ,  $B$  and  $C$  have position vectors given by

$$\vec{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane  $m$  is parallel to  $\vec{OC}$  and contains  $A$  and  $B$ .

- (i) Find the equation of  $m$ , giving your answer in the form  $ax + by + cz = d$ . [6]
- (ii) Find the length of the perpendicular from  $C$  to the line through  $A$  and  $B$ . [5]

### Q15.

- 8 Two lines have equations

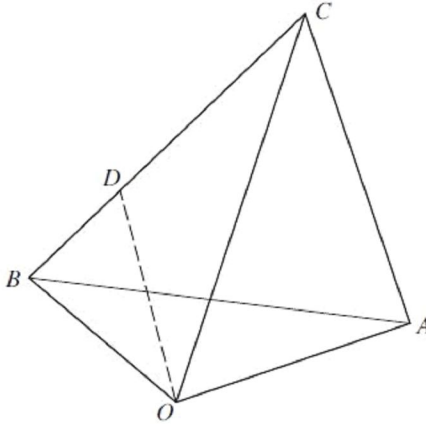
$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$$

where  $p$  is a constant. It is given that the lines intersect.

- (i) Find the value of  $p$  and determine the coordinates of the point of intersection. [5]
- (ii) Find the equation of the plane containing the two lines, giving your answer in the form  $ax + by + cz = d$ , where  $a$ ,  $b$ ,  $c$  and  $d$  are integers. [5]

**Q16.**

9



The diagram shows three points  $A$ ,  $B$  and  $C$  whose position vectors with respect to the origin  $O$  are given by  $\vec{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ ,  $\vec{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$  and  $\vec{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$ . The point  $D$  lies on  $BC$ , between  $B$  and  $C$ , and is such that  $CD = 2DB$ .

- (i) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ . [6]
- (ii) Find the position vector of  $D$ . [1]
- (iii) Show that the length of the perpendicular from  $A$  to  $OD$  is  $\frac{1}{3}\sqrt{65}$ . [4]

**Q17.**

6 Two planes have equations  $3x - y + 2z = 9$  and  $x + y - 4z = -1$ .

- (i) Find the acute angle between the planes. [3]
- (ii) Find a vector equation of the line of intersection of the planes. [6]

**Q18.**

7 The straight line  $l$  has equation  $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ . The plane  $p$  passes through the point  $(4, -1, 2)$  and is perpendicular to  $l$ .

- (i) Find the equation of  $p$ , giving your answer in the form  $ax + by + cz = d$ . [2]
- (ii) Find the perpendicular distance from the origin to  $p$ . [3]
- (iii) A second plane  $q$  is parallel to  $p$  and the perpendicular distance between  $p$  and  $q$  is 14 units. Find the possible equations of  $q$ . [3]

**Q19.**

- 10 The line  $l$  has equation  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$  and the plane  $p$  has equation  $2x + 3y - 5z = 18$ .
- (i) Find the position vector of the point of intersection of  $l$  and  $p$ . [3]
  - (ii) Find the acute angle between  $l$  and  $p$ . [4]
  - (iii) A second plane  $q$  is perpendicular to the plane  $p$  and contains the line  $l$ . Find the equation of  $q$ , giving your answer in the form  $ax + by + cz = d$ . [5]

## Q20.

- 10 The line  $l$  has equation  $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ . The point  $A$  has position vector  $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$ .
- (i) Show that the length of the perpendicular from  $A$  to  $l$  is 15. [5]
  - (ii) The line  $l$  lies in the plane with equation  $ax + by - 3z + 1 = 0$ , where  $a$  and  $b$  are constants. Find the values of  $a$  and  $b$ . [5]

## Q21.

- 7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where  $a$  is a constant.

- (i) Show that the lines intersect for all values of  $a$ . [4]
- (ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of  $a$ . [4]

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