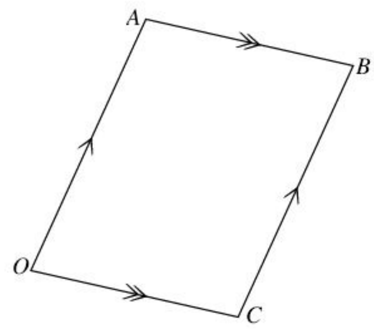
Q1.

10



The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find

(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]

(ii) the acute angle between the diagonals of the parallelogram, [5]

(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]

Q2.

6 Relative to an origin O, the position vectors of the points A, B and C are given by

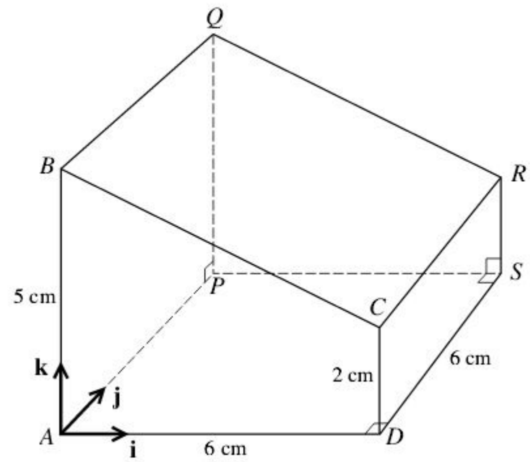
$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}, \quad \overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}.$$

(i) Use a scalar product to find angle ABC. [6]

(ii) Find the perimeter of triangle ABC, giving your answer correct to 2 decimal places. [2]

Q3.

4



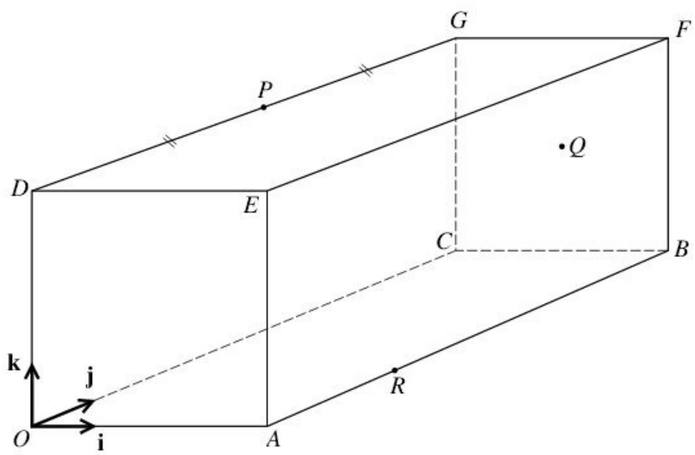
The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors **i**, **j** and **k** are parallel to AD, AP and AB respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to calculate angle PCQ. [4]

Q4.

5



In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle RQP. [4]

Q5.

6 Two vectors **u** and **v** are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.

(i) Find the values of p for which **u** is perpendicular to **v**. [3]

(ii) For the case where $p = 1$, find the angle between the directions of **u** and **v**. [4]

Q6.

2 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

(i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

(ii) the value of the constant p for which angle $BOC = 90°$. [2]

Q7.

6 Relative to an origin O, the position vectors of three points, A, B and C, are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

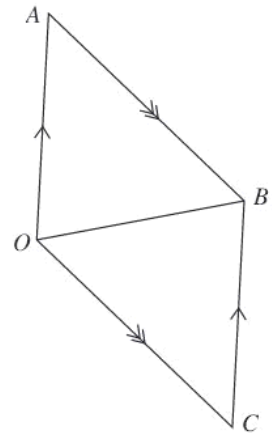
(i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q. [2]

(ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q. [2]

(iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

Q8.

8



The diagram shows a parallelogram $OABC$ in which

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

(i) Use a scalar product to find angle BOC. [6]

(ii) Find a vector which has magnitude 35 and is parallel to the vector $\overrightarrow{OC}$. [2]

Q9.

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.$$

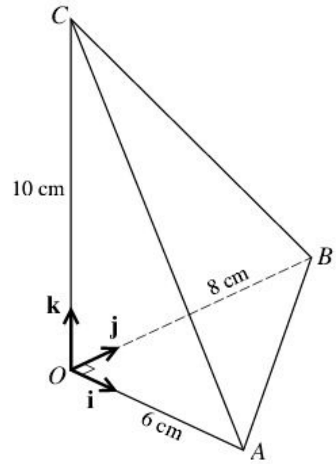
(i) Find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and has magnitude 30. [3]

(iii) Find the value of the constant p for which $\overrightarrow{OA} + p\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OC}$. [3]

Q10.

5

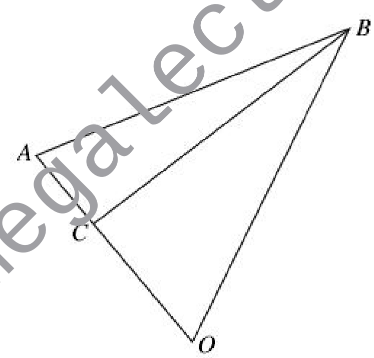


The diagram shows a pyramid $OABC$ with a horizontal base OAB where $OA = 6$ cm, $OB = 8$ cm and angle $AOB = 90°$. The point C is vertically above O and $OC = 10$ cm. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC as shown.

Use a scalar product to find angle ACB. [6]

Q11.

10



The diagram shows triangle OAB, in which the position vectors of A and B with respect to O are given by

$$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = -3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$

C is a point on OA such that $\overrightarrow{OC} = p\overrightarrow{OA}$, where p is a constant.

(i) Find angle AOB. [4]

(ii) Find $\overrightarrow{BC}$ in terms of p and vectors **i**, **j** and **k**. [1]

(iii) Find the value of p given that BC is perpendicular to OA. [4]

Q12.

8 Relative to an origin O, the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA}.\overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60°$. [4]

Q13.

6 Relative to an origin O, the position vectors of points A and B are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

(i) Use a scalar product to find angle BOA. [4]

The point C is the mid-point of AB. The point D is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

(ii) Find $\overrightarrow{DC}$. [4]

Q14.

9 The position vectors of points A and B relative to an origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. The position vectors of points C and D relative to O are $3\mathbf{a}$ and $2\mathbf{b}$ respectively. It is given that

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find the unit vector in the direction of $\overrightarrow{CD}$. [3]

(ii) The point E is the mid-point of CD. Find angle EOD. [6]

Q15.

9 The position vectors of points A and B relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} p \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix},$$

where p is a constant.

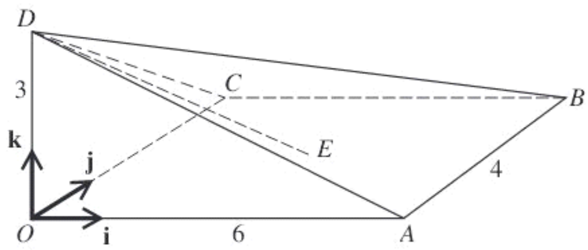
(i) In the case where OAB is a straight line, state the value of p and find the unit vector in the direction of $\overrightarrow{OA}$. [3]

(ii) In the case where OA is perpendicular to AB, find the possible values of p. [5]

(iii) In the case where $p = 3$, the point C is such that $OABC$ is a parallelogram. Find the position vector of C. [2]

Q16.

3



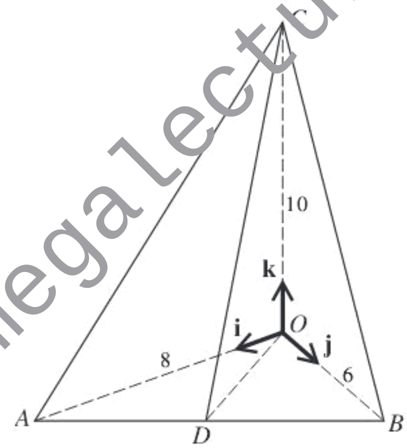
The diagram shows a pyramid $OABCD$ in which the vertical edge OD is 3 units in length. The point E is the centre of the horizontal rectangular base $OABC$. The sides OA and AB have lengths of 6 units and 4 units respectively. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Express each of the vectors $\overrightarrow{DB}$ and $\overrightarrow{DE}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to find angle BDE. [4]

Q17.

4



The diagram shows a pyramid $OABC$ in which the edge OC is vertical. The horizontal base OAB is a triangle, right-angled at O, and D is the mid-point of AB. The edges OA, OB and OC have lengths of 8 units, 6 units and 10 units respectively. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

(i) Express each of the vectors $\overrightarrow{OD}$ and $\overrightarrow{CD}$ in terms of **i**, **j** and **k**. [2]

(ii) Use a scalar product to find angle ODC. [4]