Q1.

1 The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k,

(i) $\tan(\pi - x)$, [1]

(ii) $\tan(\frac{1}{2}\pi - x)$, [1]

(iii) $\sin x$. [2]

Q2.

4 (i) Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

Q3.

5 (i) Show that the equation $2\tan^2\theta \sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta\sin^2\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

Q4.

8 (i) Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q5.

1 Solve the equation $\sin 2x = 2\cos 2x$, for $0° \leqslant x \leqslant 180°$. [4]

Q6.

1 (i) Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta\sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

Q7.

4 (i) Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]

Q8.

5 (i) Show that $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} \equiv \frac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\frac{\sin\theta}{\sin\theta + \cos\theta} + \frac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q9.

3 (i) Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \leqslant \theta \leqslant \pi$, giving solutions in terms of π. [3]

Q10.

5 (i) Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \leqslant x \leqslant 2\pi$. [4]

(ii) Hence state the number of solutions, in the interval $0 \leqslant x \leqslant 2\pi$, of the equations

(a) $2\sin 2x + 1 = 0$, [1]

(b) $\sin 2x - \cos x + 1 = 0$. [1]

Q11.

1 Solve the equation $3\tan(2x + 15°) = 4$ for $0° \leqslant x \leqslant 180°$. [4]

Q12.

2 The equation of a curve is $y = 3\cos 2x$. The equation of a line is $x + 2y = \pi$. On the same diagram, sketch the curve and the line for $0 \leqslant x \leqslant \pi$. [4]

Q13.

4 (i) Prove the identity $\frac{\sin x \tan x}{1 - \cos x} \equiv 1 + \frac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\frac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

Q14.

7 A function f is defined by $f : x \mapsto 3 - 2\tan\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $f\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Obtain an expression, in terms of x, for $f^{-1}(x)$. [3]

Q15.

3 Solve the equation $15\sin^2 x = 13 + \cos x$ for $0° \leqslant x \leqslant 180°$. [4]

Q16.

4 (i) Sketch the curve $y = 2\sin x$ for $0 \leqslant x \leqslant 2\pi$. [1]

(ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation

$$2\pi \sin x = \pi - x.$$

State the equation of the straight line. [3]

Q17.

3 (i) Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [3]

(ii) Write down the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $0 \leqslant \theta \leqslant 2\pi$. [1]

(iii) Deduce the number of roots of the equation $2\cos 2\theta - 1 = 0$ in the interval $10\pi \leqslant \theta \leqslant 20\pi$. [1]

Q18.

5 (i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$$

for $0° \leqslant \theta \leqslant 180°$. [4]

Q19.

7 (i) Solve the equation $2\cos^2\theta = 3\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [4]

(ii) The smallest positive solution of the equation $2\cos^2(n\theta) = 3\sin(n\theta)$, where n is a positive integer, is $10°$. State the value of n and hence find the largest solution of this equation in the interval $0° \leqslant \theta \leqslant 360°$. [3]

Q20.

3 Solve the equation $7\cos x + 5 = 2\sin^2 x$, for $0° \leqslant x \leqslant 360°$. [4]

Q21.

4 (i) Solve the equation $4\sin^2 x + 8\cos x - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Hence find the solution of the equation $4\sin^2\left(\frac{1}{2}\theta\right) + 8\cos\left(\frac{1}{2}\theta\right) - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [2]

Q22.

4 (i) Solve the equation $4\sin^2 x + 8\cos x - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Hence find the solution of the equation $4\sin^2\left(\frac{1}{2}\theta\right) + 8\cos\left(\frac{1}{2}\theta\right) - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [2]