These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

4 (i) Show that the equation

$$\tan(45^\circ + x) = 4\tan(45^\circ - x)$$

can be written in the form

$$3\tan^2 x - 10\tan x + 3 = 0. \qquad [4]$$

(ii) Hence solve the equation

$$\tan(45^\circ + x) = 4\tan(45^\circ - x),$$

for $0^\circ < x < 90^\circ$. [3]

Q2.

4 (i) Express $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 4\cos\theta = 4.5,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$, correct to 1 decimal place. [4]

(iii) Write down the least value of $3\sin\theta + 4\cos\theta + 7$ as θ varies. [1]

Q3.

2 (i) Prove the identity

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) \equiv (\sqrt{3})\cos x. \qquad [3]$$

(ii) Hence solve the equation

$$\cos(x + 30^\circ) + \sin(x + 60^\circ) = 1,$$

for $0^\circ < x < 90^\circ$. [2]

Q4.

5 (i) Express $5\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\cos\theta - \sin\theta = 4,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

Q5.

5 Solve the equation $\sec x = 4 - 2\tan^2 x$, giving all solutions in the interval $0° \leqslant x \leqslant 180°$. [6]

Q6.

3 (i) Show that the equation $\tan(x + 45°) = 6\tan x$ can be written in the form

$$6\tan^2 x - 5\tan x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\tan(x + 45°) = 6\tan x$, for $0° < x < 180°$. [3]

Q7.

8 (i) Express $4\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Solve the equation $4\sin\theta - 6\cos\theta = 3$ for $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Find the greatest and least possible values of $(4\sin\theta - 6\cos\theta)^2 + 8$ as θ varies. [2]

Q8.

8 (i) Express $4\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Solve the equation $4\sin\theta - 6\cos\theta = 3$ for $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Find the greatest and least possible values of $(4\sin\theta - 6\cos\theta)^2 + 8$ as θ varies. [2]

Q9.

8 (i) Prove that $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) \equiv 4\cos 2\theta$. [3]

(ii) Hence

(a) solve for $0° \leqslant \theta \leqslant 180°$ the equation $\sin^2 2\theta(\operatorname{cosec}^2\theta - \sec^2\theta) = 3$, [4]

(b) find the exact value of $\operatorname{cosec}^2 15° - \sec^2 15°$. [2]

Q10.

4 (i) Given that $35 + \sec^2\theta = 12\tan\theta$, find the value of $\tan\theta$. [3]

(ii) Hence, showing the use of an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45°)$, [2]

(b) $\tan 2\theta$. [2]

Q11.

4 (i) Express $9\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation $9\sin\theta - 12\cos\theta = 4$ for $0° \leqslant \theta \leqslant 360°$, [4]

(iii) state the largest value of k for which the equation $9\sin\theta - 12\cos\theta = k$ has any solutions. [1]

Q12.

7 (i) Express $5\sin 2\theta + 2\cos 2\theta$ in the form $R\sin(2\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$5\sin 2\theta + 2\cos 2\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$, [5]

(iii) determine the least value of $\dfrac{1}{(10\sin 2\theta + 4\cos 2\theta)^2}$ as θ varies. [2]

Q13.

8 (i) Prove the identity

$$\frac{1}{\sin(x - 60°) + \cos(x - 30°)} \equiv \operatorname{cosec} x.$$ [3]

(ii) Hence solve the equation

$$\frac{2}{\sin(x - 60°) + \cos(x - 30°)} = 3\cot^2 x - 2,$$

for $0° < x < 360°$. [6]

Q14.

5 The angle x, measured in degrees, satisfies the equation

$$\cos(x - 30^\circ) = 3\sin(x - 60^\circ).$$

(i) By expanding each side, show that the equation may be simplified to

$$(2\sqrt{3})\cos x = \sin x.$$ [3]

(ii) Find the two possible values of x lying between 0° and 360°. [3]

(iii) Find the exact value of $\cos 2x$, giving your answer as a fraction. [3]

Q15.

4 (i) Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of α. [3]

(ii) Hence show that one solution of the equation

$$\cos\theta + (\sqrt{3})\sin\theta = \sqrt{2}$$

is $\theta = \frac{7}{12}\pi$, and find the other solution in the interval $0 < \theta < 2\pi$. [4]

Q16.

3 Find the values of x satisfying the equation

$$3\sin 2x = \cos x,$$

for $0^\circ \leqslant x \leqslant 90^\circ$. [4]

Q17.

8 (i) Express $\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

$$\frac{1}{(\cos\theta + \sin\theta)^2} = \frac{1}{2}\sec^2(\theta - \tfrac{1}{4}\pi).$$ [1]

(iii) By differentiating $\frac{\sin x}{\cos x}$, show that if $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$. [3]

(iv) Using the results of parts **(ii)** and **(iii)**, show that

$$\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + \sin\theta)^2}\,d\theta = 1.$$ [3]

Q18.

3 (i) Express $12\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$12\cos\theta - 5\sin\theta = 10,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

Q19.

4 (i) Prove the identity

$$\tan(x + 45°) - \tan(45° - x) \equiv 2\tan 2x.$$ [4]

(ii) Hence solve the equation

$$\tan(x + 45°) - \tan(45° - x) = 2,$$

for $0° \leqslant x \leqslant 180°$. [3]

Q20.

6 (i) Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\sin\theta - 15\cos\theta = 14,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

Q21.

4 (i) Show that the equation

$$\sin(x + 30°) = 2\cos(x + 60°)$$

can be written in the form

$$(3\sqrt{3})\sin x = \cos x.$$ [3]

(ii) Hence solve the equation

$$\sin(x + 30°) = 2\cos(x + 60°),$$

for $-180° \leqslant x \leqslant 180°$. [3]

Q22.

4 (i) Show that the equation $\sin(60° - x) = 2\sin x$ can be written in the form $\tan x = k$, where k is a constant. [4]

(ii) Hence solve the equation $\sin(60° - x) = 2\sin x$, for $0° < x < 360°$. [2]

Q23.

6 (i) Express $3\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\cos x + 4\sin x = 4.5,$$

giving all solutions in the interval $0° < x < 360°$. [4]

Q24.

5 Solve the equation $8 + \cot\theta = 2\operatorname{cosec}^2\theta$, giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

Q25.

6 (i) Express $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$2\sin\theta - \cos\theta = -0.4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

Q26.

8 (i) Express $5\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\cos\theta - 3\sin\theta = 4,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

(iii) Write down the least value of $15\cos\theta - 9\sin\theta$ as θ varies. [1]

Q27.

5 Solve the equation $5\sec^2 2\theta = \tan 2\theta + 9$, giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [6]

Q28.

3 Solve the equation

$$2\cos 2\theta = 4\cos\theta - 3,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

Q29.

8 (a) Given that $\tan A = t$ and $\tan(A + B) = 4$, find $\tan B$ in terms of t. [3]

(b) Solve the equation

$$2\tan(45° - x) = 3\tan x,$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. [6]

Q30.

7 (i) Express $3\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\cos 2x + \sin 2x = 2,$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. [5]

Q31.

3 Solve the equation $2\cot^2\theta - 5\operatorname{cosec}\theta = 10$, giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

Q32.

2 Solve the equation $3\sin 2\theta \tan\theta = 2$ for $0° < \theta < 180°$. [4]

Q33.

7 The angle α lies between $0°$ and $90°$ and is such that

$$2\tan^2\alpha + \sec^2\alpha = 5 - 4\tan\alpha.$$

(i) Show that

$$3\tan^2\alpha + 4\tan\alpha - 4 = 0$$

and hence find the exact value of $\tan\alpha$. [4]

(ii) It is given that the angle β is such that $\cot(\alpha + \beta) = 6$. Without using a calculator, find the exact value of $\cot\beta$. [5]

Q34.

7 (i) Express $5\cos\theta - 12\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $5\cos\theta - 12\sin\theta = 8$ for $0° < \theta < 360°$. [4]

(iii) Find the greatest possible value of

$$7 + 5\cos\tfrac{1}{2}\phi - 12\sin\tfrac{1}{2}\phi$$

as ϕ varies, and determine the smallest positive value of ϕ for which this greatest value occurs. [4]

P3 (variant1 and 3)

Q1.

2 Solve the equation

$$\sin\theta = 2\cos 2\theta + 1,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

Q2.

3 Solve the equation

$$\tan(45° - x) = 2\tan x,$$

giving all solutions in the interval $0° < x < 180°$. [5]

Q3.

9 (i) Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4\theta \, d\theta$. [3]

Q4.

4 (i) Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2\theta) = k(1 - 3\tan^2\theta).$$ [4]

(ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

Q5.

6 It is given that $\tan 3x = k \tan x$, where k is a constant and $\tan x \neq 0$.

(i) By first expanding $\tan(2x + x)$, show that

$$(3k - 1)\tan^2 x = k - 3.$$ [4]

(ii) Hence solve the equation $\tan 3x = k \tan x$ when $k = 4$, giving all solutions in the interval $0^\circ < x < 180^\circ$. [3]

(iii) Show that the equation $\tan 3x = k \tan x$ has no root in the interval $0^\circ < x < 180^\circ$ when $k = 2$. [1]

Q6.

9 (i) Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\int \frac{50}{(4\cos\theta + 3\sin\theta)^2} \, d\theta$. [3]

Q7.

3 Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

Q8.

3 Solve the equation

$$\cos(\theta + 60°) = 2\sin\theta,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [5]

Q9.

8 (i) Express $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

(a) $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta = -4$, [2]

(b) $(\sqrt{6})\cos\frac{1}{2}\theta + (\sqrt{10})\sin\frac{1}{2}\theta = 3$. [4]

Q10.

6 (i) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0° < \theta < 90°$. [5]

Q11.

3 (i) Express $8\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $8\cos\theta + 15\sin\theta = 12$, giving all solutions in the interval $0° < \theta < 360°$. [4]

Q12.

3 Solve the equation

$$\sin(\theta + 45°) = 2\cos(\theta - 30°),$$

giving all solutions in the interval $0° < \theta < 180°$. [5]

Q13.

2 (i) Express $24\sin\theta - 7\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence find the smallest positive value of θ satisfying the equation

$$24\sin\theta - 7\cos\theta = 17.$$ [2]

Q14.

7 (i) Given that $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$, show that $2\sin\theta + 4\cos\theta = 3$. [3]

(ii) Express $2\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(iii) Hence solve the equation $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$ for $0° < \theta < 360°$. [4]

Q15.

1 (i) Simplify $\sin 2\alpha \sec\alpha$. [2]

(ii) Given that $3\cos 2\beta + 7\cos\beta = 0$, find the exact value of $\cos\beta$. [3]

Q16.

3 (i) Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

Q17.

8 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \dfrac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \tfrac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

Q18.

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]