

These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

- 5 (i) By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has exactly one root. [3]

- (ii) Verify by calculation that the root lies between 1.0 and 1.4. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

to determine the root correct to 2 decimal places, showing the result of each iteration. [3]

Q2.

- 2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{5} \left(4x_n + \frac{306}{x_n^4} \right),$$

with initial value $x_1 = 3$, converges to α .

- (i) Use this iterative formula to find α correct to 3 decimal places, showing the result of each iteration. [3]

- (ii) State an equation satisfied by α , and hence show that the exact value of α is $\sqrt[3]{306}$. [2]

Q3.

- 3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{2}{x_n^3},$$

with initial value $x_1 = 2$, converges to α .

- (i) Use this iteration to calculate α correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

- (ii) State an equation which is satisfied by α and hence find the exact value of α . [2]

Q4.

- 6 (i) By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation $x = 9e^{-2x}$. [2]
- (ii) Verify, by calculation, that this root lies between 1 and 2. [2]
- (iii) Show that, if a sequence of values given by the iterative formula
- $$x_{n+1} = \frac{1}{2}(\ln 9 - \ln x_n)$$
- converges, then it converges to the root of the equation given in part (i). [2]
- (iv) Use the iterative formula, with $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q5.

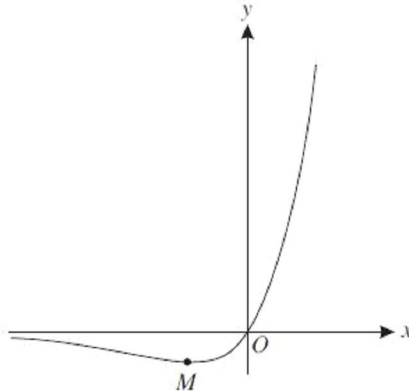
- 5 (i) By sketching a suitable pair of graphs, show that the equation
- $$\sec x = 3 - x,$$
- where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]
- (ii) Verify by calculation that this root lies between 1.0 and 1.2. [2]
- (iii) Show that this root also satisfies the equation
- $$x = \cos^{-1}\left(\frac{1}{3-x}\right).$$
- [1]
- (iv) Use the iterative formula
- $$x_{n+1} = \cos^{-1}\left(\frac{1}{3-x_n}\right),$$
- with initial value $x_1 = 1.1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q6.

- 8 The constant a , where $a > 1$, is such that $\int_1^a \left(x + \frac{1}{x}\right) dx = 6$.
- (i) Find an equation satisfied by a , and show that it can be written in the form
- $$a = \sqrt{13 - 2 \ln a}.$$
- [5]
- (ii) Verify, by calculation, that the equation $a = \sqrt{13 - 2 \ln a}$ has a root between 3 and 3.5. [2]
- (iii) Use the iterative formula
- $$a_{n+1} = \sqrt{13 - 2 \ln a_n},$$
- with $a_1 = 3.2$, to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q7.

7



The diagram shows the curve $y = xe^{2x}$ and its minimum point M .

(i) Find the exact coordinates of M . [5]

(ii) Show that the curve intersects the line $y = 20$ at the point whose x -coordinate is the root of the equation

$$x = \frac{1}{2} \ln\left(\frac{20}{x}\right). \quad [1]$$

(iii) Use the iterative formula

$$x_{n+1} = \frac{1}{2} \ln\left(\frac{20}{x_n}\right),$$

with initial value $x_1 = 1.3$, to calculate the root correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

Q8.

7 (i) By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 2 - x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 0$ and $x = 0.5$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \ln(2 - x_n)$$

converges, then it converges to the root of the equation in part (i). [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q9.

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has only one root. [2]

- (ii) Verify by calculation that this root lies between $x = 1.3$ and $x = 1.4$. [2]

- (iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{2 - \ln x_n}$$

converges, then it converges to the root of the equation in part (i). [1]

- (iv) Use the iterative formula $x_{n+1} = \sqrt{2 - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q10.

- 7 (i) By sketching a suitable pair of graphs, show that the equation

$$e^{2x} = 14 - x^2$$

has exactly two real roots. [3]

- (ii) Show by calculation that the positive root lies between 1.2 and 1.3. [2]

- (iii) Show that this root also satisfies the equation

$$x = \frac{1}{2} \ln(14 - x^2). \quad [1]$$

- (iv) Use an iteration process based on the equation in part (iii), with a suitable starting value, to find the root correct to 2 decimal places. Give the result of each step of the process to 4 decimal places. [3]

Q11.

- 3 The sequence x_1, x_2, x_3, \dots defined by

$$x_1 = 1, \quad x_{n+1} = \frac{1}{2} \sqrt[3]{(x_n^2 + 6)}$$

converges to the value α .

- (i) Find the value of α correct to 3 decimal places. Show your working, giving each calculated value of the sequence to 5 decimal places. [3]
- (ii) Find, in the form $ax^3 + bx^2 + c = 0$, an equation of which α is a root. [2]

Q12.

6 A curve has parametric equations

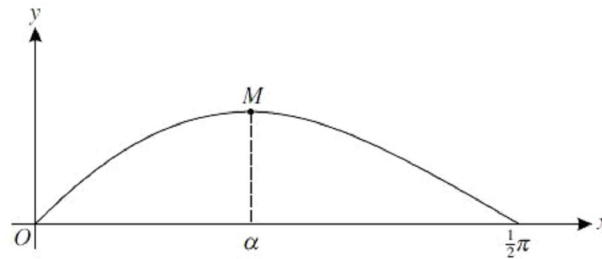
$$x = \frac{1}{(2t+1)^2}, \quad y = \sqrt{t+2}.$$

The point P on the curve has parameter p and it is given that the gradient of the curve at P is -1 .

- (i) Show that $p = (p+2)^{\frac{1}{5}} - \frac{1}{2}$. [6]
- (ii) Use an iterative process based on the equation in part (i) to find the value of p correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places. [3]

Q13.

6



The diagram shows the curve $y = \frac{\sin 2x}{x+2}$ for $0 \leq x \leq \frac{1}{2}\pi$. The x -coordinate of the maximum point M is denoted by α .

- (i) Find $\frac{dy}{dx}$ and show that α satisfies the equation $\tan 2x = 2x + 4$. [4]
- (ii) Show by calculation that α lies between 0.6 and 0.7. [2]
- (iii) Use the iterative formula $x_{n+1} = \frac{1}{2} \tan^{-1}(2x_n + 4)$ to find the value of α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q14.

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 4x - 2,$$

where x is in radians, has only one root for $0 \leq x \leq \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.9$. [2]

- (iii) Show that this root also satisfies the equation

$$x = \frac{1 + 2 \tan x}{4 \tan x}. \quad [1]$$

- (iv) Use the iterative formula $x_{n+1} = \frac{1 + 2 \tan x_n}{4 \tan x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q15.

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$3e^x = 8 - 2x$$

has only one root. [2]

- (ii) Verify by calculation that this root lies between $x = 0.7$ and $x = 0.8$. [2]

- (iii) Show that this root also satisfies the equation

$$x = \ln \left(\frac{8 - 2x}{3} \right). \quad [1]$$

- (iv) Use the iterative formula $x_{n+1} = \ln \left(\frac{8 - 2x_n}{3} \right)$ to determine this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q16.

- 4 (i) By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\sin x = \frac{1}{x^2}. \quad [2]$$

- (ii) Verify by calculation that this root lies between 1 and 1.5. [2]

- (iii) Show that this value of x is also a root of the equation

$$x = \sqrt{(\operatorname{cosec} x)}. \quad [1]$$

- (iv) Use the iterative formula

$$x_{n+1} = \sqrt{(\operatorname{cosec} x_n)}$$

to determine this root correct to 3 significant figures, showing the value of each approximation that you calculate. [3]

Q17.

- 5 (i) By sketching a suitable pair of graphs, for $x < 0$, show that exactly one root of the equation $x^2 = 2^x$ is negative. [2]

- (ii) Verify by calculation that this root lies between -1.0 and -0.5 . [2]

- (iii) Use the iterative formula

$$x_{n+1} = -\sqrt{(2^{x_n})}$$

to determine this root correct to 2 significant figures, showing the result of each iteration. [3]

Q18.

- 6 (i) By sketching a suitable pair of graphs, show that there is only one value of x in the interval $0 < x < \frac{1}{2}\pi$ that is a root of the equation

$$\cot x = x. \quad [2]$$

- (ii) Verify by calculation that this root lies between 0.8 and 0.9 radians. [2]

- (iii) Show that this value of x is also a root of the equation

$$x = \tan^{-1}\left(\frac{1}{x}\right). \quad [1]$$

- (iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{x_n}\right)$$

to determine this root correct to 2 decimal places, showing the result of each iteration. [3]

Q19.

- 5 (i) By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation

$$\frac{1}{x} = \ln x. \quad [2]$$

- (ii) Verify by calculation that this root lies between 1 and 2. [2]

- (iii) Show that this root also satisfies the equation

$$x = e^{\frac{1}{x}}. \quad [1]$$

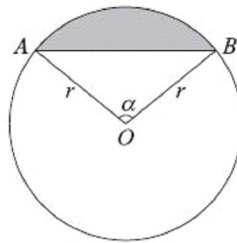
- (iv) Use the iterative formula

$$x_{n+1} = e^{\frac{1}{x_n}},$$

with initial value $x_1 = 1.8$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q20.

5



The diagram shows a chord joining two points, A and B , on the circumference of a circle with centre O and radius r . The angle AOB is α radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

- (i) Show that α satisfies the equation

$$x = \frac{1}{3}\pi + \sin x. \quad [3]$$

- (ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q21.

- 2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n}{3} + \frac{4}{x_n^2},$$

with initial value $x_1 = 2$, converges to α .

- (i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]
- (ii) State an equation that is satisfied by α and hence find the exact value of α . [2]

Q22.

- 7 (i) By sketching a suitable pair of graphs, show that the equation

$$\cos x = 2 - 2x,$$

where x is in radians, has only one root for $0 \leq x \leq \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between 0.5 and 1. [2]

- (iii) Show that, if a sequence of values given by the iterative formula

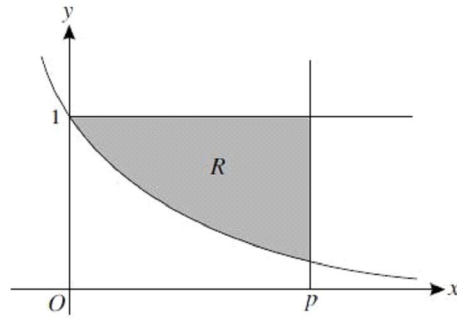
$$x_{n+1} = 1 - \frac{1}{2} \cos x_n$$

converges, then it converges to the root of the equation in part (i). [1]

- (iv) Use this iterative formula, with initial value $x_1 = 0.6$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q23.

7



The diagram shows the curve $y = e^{-x}$. The shaded region R is bounded by the curve and the lines $y = 1$ and $x = p$, where p is a constant.

(i) Find the area of R in terms of p . [4]

(ii) Show that if the area of R is equal to 1 then

$$p = 2 - e^{-p}. \quad [1]$$

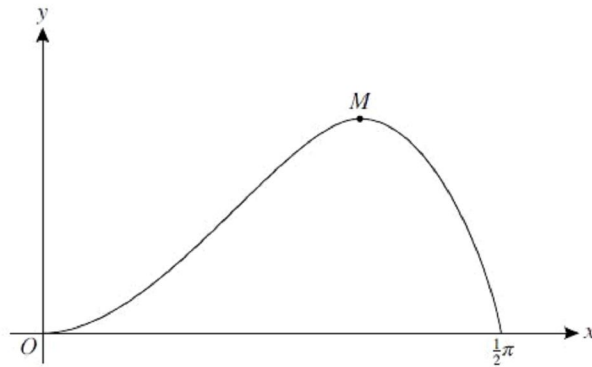
(iii) Use the iterative formula

$$p_{n+1} = 2 - e^{-p_n},$$

with initial value $p_1 = 2$, to calculate the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q24.

7



The diagram shows the curve $y = x^2 \cos x$, for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (i) Show by differentiation that the x -coordinate of M satisfies the equation

$$\tan x = \frac{2}{x}. \quad [4]$$

- (ii) Verify by calculation that this equation has a root (in radians) between 1 and 1.2. [2]

- (iii) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{2}{x_n}\right)$ to determine this root correct to 2 decimal places.
Give the result of each iteration to 4 decimal places. [3]

Q25.

- 6 The curve with equation $y = \frac{6}{x^2}$ intersects the line $y = x + 1$ at the point P .

- (i) Verify by calculation that the x -coordinate of P lies between 1.4 and 1.6. [2]

- (ii) Show that the x -coordinate of P satisfies the equation

$$x = \sqrt{\left(\frac{6}{x+1}\right)}. \quad [2]$$

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt{\left(\frac{6}{x_n+1}\right)},$$

with initial value $x_1 = 1.5$, to determine the x -coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q26.

- 2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{7x_n}{8} + \frac{5}{2x_n^4},$$

with initial value $x_1 = 1.7$, converges to α .

- (i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]
- (ii) State an equation that is satisfied by α and hence show that $\alpha = \sqrt[3]{20}$. [2]

Q27.

- 6 (i) Verify by calculation that the cubic equation

$$x^3 - 2x^2 + 5x - 3 = 0$$

has a root that lies between $x = 0.7$ and $x = 0.8$. [2]

- (ii) Show that this root also satisfies an equation of the form

$$x = \frac{ax^2 + 3}{x^2 + b},$$

where the values of a and b are to be found. [2]

- (iii) With these values of a and b , use the iterative formula

$$x_{n+1} = \frac{ax_n^2 + 3}{x_n^2 + b}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q28.

- 5 (i) By sketching a suitable pair of graphs, show that the equation

$$\frac{1}{x} = \sin x,$$

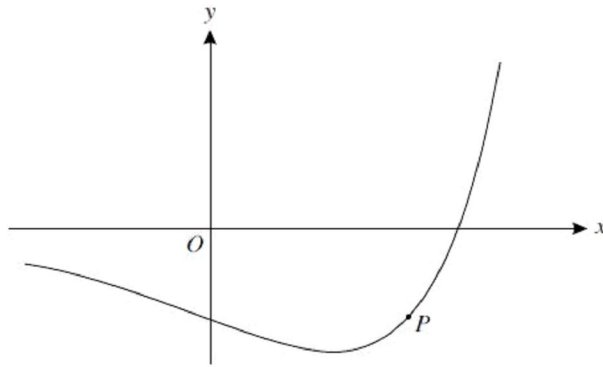
where x is in radians, has only one root for $0 < x \leq \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between $x = 1.1$ and $x = 1.2$. [2]

- (iii) Use the iterative formula $x_{n+1} = \frac{1}{\sin x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q29.

7



The diagram shows the curve $y = (x - 4)e^{\frac{1}{2}x}$. The curve has a gradient of 3 at the point P .

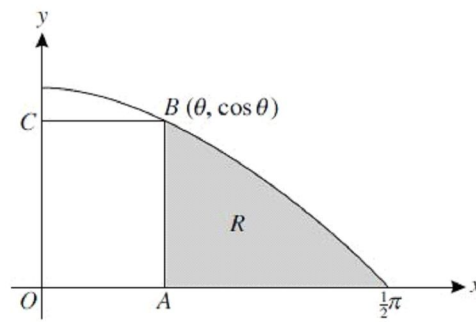
- (i) Show that the x -coordinate of P satisfies the equation

$$x = 2 + 6e^{-\frac{1}{2}x}. \quad [4]$$

- (ii) Verify that the equation in part (i) has a root between $x = 3.1$ and $x = 3.3$. [2]
- (iii) Use the iterative formula $x_{n+1} = 2 + 6e^{-\frac{1}{2}x_n}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q30.

5



The diagram shows the curve $y = \cos x$, for $0 \leq x \leq \frac{1}{2}\pi$. A rectangle $OABC$ is drawn, where B is the point on the curve with x -coordinate θ , and A and C are on the axes, as shown. The shaded region R is bounded by the curve and by the lines $x = \theta$ and $y = 0$.

(i) Find the area of R in terms of θ . [2]

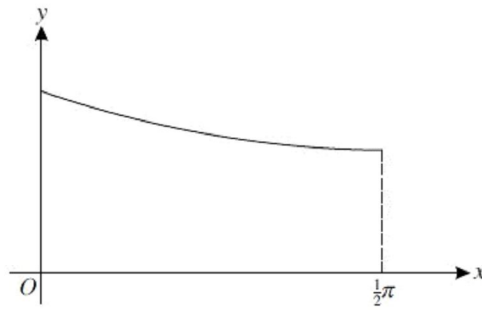
(ii) The area of the rectangle $OABC$ is equal to the area of R . Show that

$$\theta = \frac{1 - \sin \theta}{\cos \theta}. \quad [1]$$

(iii) Use the iterative formula $\theta_{n+1} = \frac{1 - \sin \theta_n}{\cos \theta_n}$, with initial value $\theta_1 = 0.5$, to determine the value of θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q31.

4



The diagram shows the part of the curve $y = \sqrt{2 - \sin x}$ for $0 \leq x \leq \frac{1}{2}\pi$.

- (i) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \sqrt{2 - \sin x} \, dx,$$

giving your answer correct to 2 decimal places.

[3]

- (ii) The line $y = x$ intersects the curve $y = \sqrt{2 - \sin x}$ at the point P . Use the iterative formula

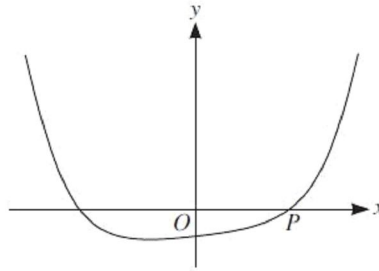
$$x_{n+1} = \sqrt{2 - \sin x_n}$$

to determine the x -coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

Q32.

2



The diagram shows the curve $y = x^4 + 2x - 9$. The curve cuts the positive x -axis at the point P .

(i) Verify by calculation that the x -coordinate of P lies between 1.5 and 1.6. [2]

(ii) Show that the x -coordinate of P satisfies the equation

$$x = \sqrt[3]{\left(\frac{9}{x} - 2\right)}. \quad [1]$$

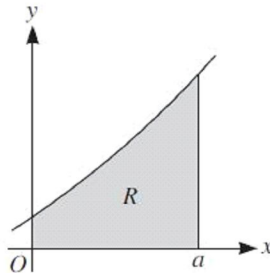
(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\left(\frac{9}{x_n} - 2\right)}$$

to determine the x -coordinate of P correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q33.

7



The diagram shows part of the curve $y = 8x + \frac{1}{2}e^x$. The shaded region R is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = a$, where a is positive. The area of R is equal to $\frac{1}{2}$.

(i) Find an equation satisfied by a , and show that the equation can be written in the form

$$a = \sqrt{\left(\frac{2 - e^a}{8}\right)}. \quad [5]$$

(ii) Verify by calculation that the equation $a = \sqrt{\left(\frac{2 - e^a}{8}\right)}$ has a root between 0.2 and 0.3. [2]

(iii) Use the iterative formula $a_{n+1} = \sqrt{\left(\frac{2 - e^{a_n}}{8}\right)}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q34.

- 4 (i) By sketching a suitable pair of graphs, show that the equation

$$3 \ln x = 15 - x^3$$

has exactly one real root. [3]

- (ii) Show by calculation that the root lies between 2.0 and 2.5. [2]
- (iii) Use the iterative formula $x_{n+1} = \sqrt[3]{15 - 3 \ln x_n}$ to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q35.

- 1 (i) Solve the equation $|x + 2| = |x - 13|$. [2]
- (ii) Hence solve the equation $|3^y + 2| = |3^y - 13|$, giving your answer correct to 3 significant figures. [2]

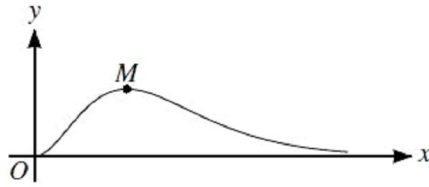
Q36.

- 7 It is given that $\int_0^a \left(\frac{1}{2}e^{3x} + x^2\right) dx = 10$, where a is a positive constant.

- (i) Show that $a = \frac{1}{3} \ln(61 - 2a^3)$. [4]
- (ii) Show by calculation that the value of a lies between 1.0 and 1.5. [2]
- (iii) Use an iterative formula, based on the equation in part (i), to find the value of a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q37.

6



The diagram shows part of the curve $y = \frac{x^2}{1 + e^{3x}}$ and its maximum point M . The x -coordinate of M is denoted by m .

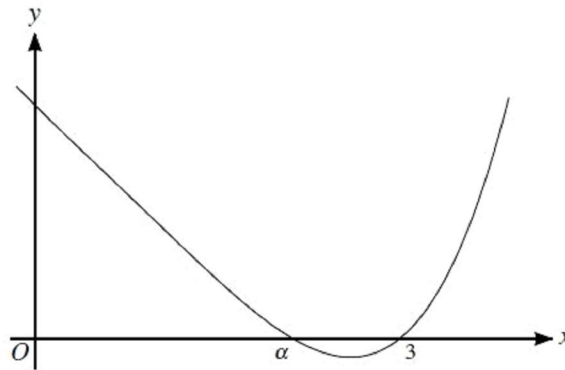
- (i) Find $\frac{dy}{dx}$ and hence show that m satisfies the equation $x = \frac{2}{3}(1 + e^{-3x})$. [4]
- (ii) Show by calculation that m lies between 0.7 and 0.8. [2]
- (iii) Use an iterative formula based on the equation in part (i) to find m correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Q38.

- 1 Solve the equation $|3x - 1| = |2x + 5|$. [3]

Q39.

6



The polynomial $p(x)$ is defined by

$$p(x) = x^4 - 3x^3 + 3x^2 - 25x + 48.$$

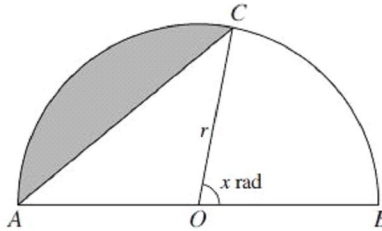
The diagram shows the curve $y = p(x)$ which crosses the x -axis at $(\alpha, 0)$ and $(3, 0)$.

- (i) Divide $p(x)$ by a suitable linear factor and hence show that α is a root of the equation $x = \sqrt[3]{16 - 3x}$. [5]
- (ii) Use the iterative formula $x_{n+1} = \sqrt[3]{16 - 3x_n}$ to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

P3 (variant1 and 3)

Q1.

6



The diagram shows a semicircle ACB with centre O and radius r . The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

- (i) Show that x satisfies the equation

$$x = \frac{3}{4}\pi - \sin x. \quad [3]$$

- (ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \frac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q2.

- 6 The curve $y = \frac{\ln x}{x+1}$ has one stationary point.

- (i) Show that the x -coordinate of this point satisfies the equation

$$x = \frac{x+1}{\ln x},$$

and that this x -coordinate lies between 3 and 4. [5]

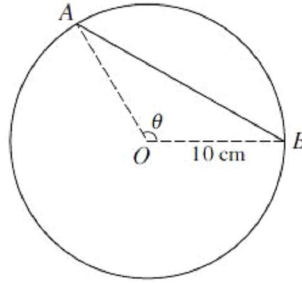
- (ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x -coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q3.

6



The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio $1 : 4$ and it is required to find the length of AB . The angle AOB is θ radians.

- (i) Show that $\theta = \frac{2}{5}\pi + \sin \theta$. [3]
- (ii) Showing all your working, use an iterative formula, based on the equation in part (i), with an initial value of 2.1 , to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

Q4.

- 6 (i) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between 0.5 and 0.8 . [2]
- (iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1+x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q5.

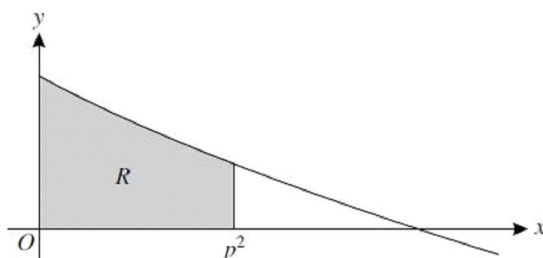
- 1 Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

Q6.

- 10 (i) It is given that $2 \tan 2x + 5 \tan^2 x = 0$. Denoting $\tan x$ by t , form an equation in t and hence show that either $t = 0$ or $t = \sqrt[3]{t + 0.8}$. [4]
- (ii) It is given that there is exactly one real value of t satisfying the equation $t = \sqrt[3]{t + 0.8}$. Verify by calculation that this value lies between 1.2 and 1.3. [2]
- (iii) Use the iterative formula $t_{n+1} = \sqrt[3]{t_n + 0.8}$ to find the value of t correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]
- (iv) Using the values of t found in previous parts of the question, solve the equation
- $$2 \tan 2x + 5 \tan^2 x = 0$$
- for $-\pi \leq x \leq \pi$. [3]

Q7.

7



The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geq 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R . The area of R is equal to 1.

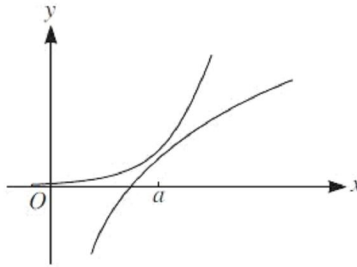
- (i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x}) \, dx$. Hence show that $\sin p = \frac{3 - 2 \cos p}{2p}$. [6]
- (ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\frac{3 - 2 \cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q8.

- 4 (i) Solve the equation $|4x - 1| = |x - 3|$. [3]
- (ii) Hence solve the equation $|4^{y+1} - 1| = |4^y - 3|$ correct to 3 significant figures. [3]

Q9.

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2 \ln x$. When $x = a$ the tangents to the curves are parallel.

- (i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]
- (ii) Verify by calculation that this equation has a root between 1 and 2. [2]
- (iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

Q10.

- 3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{15}{x_n^3},$$

with initial value $x_1 = 3$, converges to α .

- (i) Use this iterative formula to find α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]
- (ii) State an equation satisfied by α and hence find the exact value of α . [2]

Q11.

- 4 (i) By sketching suitable graphs, show that the equation

$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (ii) Verify by calculation that this root lies between 0.6 and 1. [2]
- (iii) Use the iterative formula

$$x_{n+1} = \frac{1}{2}\sqrt{1 + \cot x_n}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q12.

- 7 (i) Given that $\int_1^a \frac{\ln x}{x^2} dx = \frac{2}{5}$, show that $a = \frac{5}{3}(1 + \ln a)$. [5]
- (ii) Use an iteration formula based on the equation $a = \frac{5}{3}(1 + \ln a)$ to find the value of a correct to 2 decimal places. Use an initial value of 4 and give the result of each iteration to 4 decimal places. [3]

Q13.

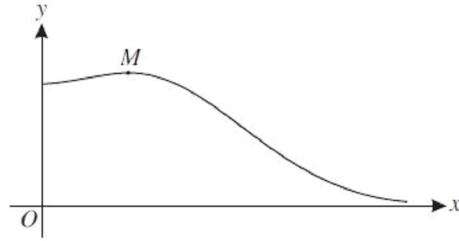
- 5 (i) By sketching a suitable pair of graphs, show that the equation
- $$\sec x = 3 - \frac{1}{2}x^2,$$
- where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]
- (ii) Verify by calculation that this root lies between 1 and 1.4. [2]
- (iii) Show that this root also satisfies the equation
- $$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right). [1]$$
- (iv) Use an iterative formula based on the equation in part (iii) to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q14.

- 5 It is given that $\int_1^a x \ln x dx = 22$, where a is a constant greater than 1.
- (i) Show that $a = \sqrt{\left(\frac{87}{2 \ln a - 1}\right)}$. [5]
- (ii) Use an iterative formula based on the equation in part (i) to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

Q15.

8



The diagram shows the curve $y = e^{-\frac{1}{2}x^2} \sqrt{1 + 2x^2}$ for $x \geq 0$, and its maximum point M .

(i) Find the exact value of the x -coordinate of M . [4]

(ii) The sequence of values given by the iterative formula

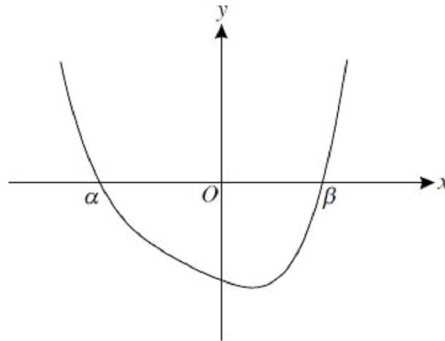
$$x_{n+1} = \sqrt{(\ln(4 + 8x_n^2))},$$

with initial value $x_1 = 2$, converges to a certain value α . State an equation satisfied by α and hence show that α is the x -coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q16.

6



The diagram shows the curve $y = x^4 + 2x^3 + 2x^2 - 4x - 16$, which crosses the x -axis at the points $(\alpha, 0)$ and $(\beta, 0)$ where $\alpha < \beta$. It is given that α is an integer.

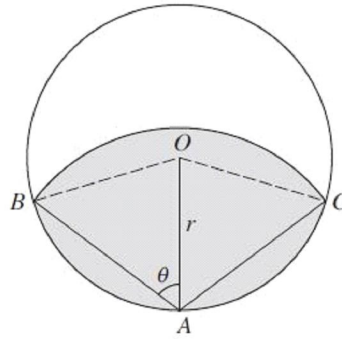
(i) Find the value of α . [2]

(ii) Show that β satisfies the equation $x = \sqrt[3]{(8 - 2x)}$. [3]

(iii) Use an iteration process based on the equation in part (ii) to find the value of β correct to 2 decimal places. Show the result of each iteration to 4 decimal places. [3]

Q17.

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r . A circular arc with centre A meets the circumference at B and C . The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C . The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \frac{2 \sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2} \cos^{-1} \left(\frac{2 \sin 2\theta_n - \pi}{4\theta_n} \right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

Q18.

5 It is given that $\int_0^p 4xe^{-\frac{1}{2}x} dx = 9$, where p is a positive constant.

(i) Show that $p = 2 \ln \left(\frac{8p + 16}{7} \right)$. [5]

(ii) Use an iterative process based on the equation in part (i) to find the value of p correct to 3 significant figures. Use a starting value of 3.5 and give the result of each iteration to 5 significant figures. [3]

Q19.

- 8 (i) By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation
- $$\operatorname{cosec} x = x(\pi - x)$$
- has exactly two real roots in the interval $0 < x < \pi$. [3]
- (ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \frac{1 + x^2 \sin x}{\pi \sin x}$. [2]
- (iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β , where $\alpha < \beta$.
- (a) Use the iterative formula
- $$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$
- to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]
- (b) Deduce the value of β correct to 2 decimal places. [1]

Q20.

- 4 The equation $x = \frac{10}{e^{2x} - 1}$ has one positive real root, denoted by α .
- (i) Show that α lies between $x = 1$ and $x = 2$. [2]
- (ii) Show that if a sequence of positive values given by the iterative formula
- $$x_{n+1} = \frac{1}{2} \ln \left(1 + \frac{10}{x_n} \right)$$
- converges, then it converges to α . [2]
- (iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q21.

- 6 It is given that $\int_1^a \ln(2x) \, dx = 1$, where $a > 1$.
- (i) Show that $a = \frac{1}{2} \exp \left(1 + \frac{\ln 2}{a} \right)$, where $\exp(x)$ denotes e^x . [6]
- (ii) Use the iterative formula
- $$a_{n+1} = \frac{1}{2} \exp \left(1 + \frac{\ln 2}{a_n} \right)$$
- to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Q22.

- 9 (i) Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

- (ii) Verify by calculation that the root lies between 3 and 4. [2]

- (iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(40 - \ln(x_n + 1))},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

- (iv) Deduce the root of the equation

$$(e^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

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