Q1.

4 $y = 6x - x^2$ Meets $y = 5$ when $x = 1$ or $x = 5$. Integral = $3x^2 - \frac{1}{3}x^3$ Their limits (1 to 5) used → 30⅔ Area of rectangle = 20 Shaded area = 10⅔ (integral of $6x - x^2 - 5$ B1, M1, A1 DM1 as above, then "−5x" B1√ A1)	B1 M1 A1 DM1 B1√ A1 [6]	co attempt to integrate. co. value at top limit − value at lower co to his x values co

Q2.

5 $\frac{dy}{dx} = \frac{6}{\sqrt{3x-2}}$ **(i)** $x = 2$, tangent has gradient 3 → normal has gradient $-\frac{1}{3}$ → $y - 11 = -\frac{1}{3}(x - 2)$	M1 M1 A1 [3]	Use of $m_1m_2 = -1$ with dy/dx Correct form of line eqn. for normal
(ii) Integrate → $6\frac{\sqrt{3x-2}}{\frac{1}{2}} \div 3$ → $y = 4\sqrt{3x-2} + c$ through (2,11) → $y = 4\sqrt{3x-2} + 3$	B1 B1 M1 A1 [4]	Without the ÷3 For ÷3, even if B0 above Using (2, 11) for c co

Q3.

9 $y = x + \frac{4}{x}$		
(i) $x + \frac{4}{x} = 5 \rightarrow A(1, 5), B(4, 5)$	B1 B1	co. co.
$\frac{dy}{dx} = 1 - \frac{4}{x^2}$	M1	Differentiates.
$= 0$ when $x = 2$, $M(2, 4)$.	DM1 A1 [5]	Setting to 0. co.
(ii) Vol of cylinder $= \pi 5^2.3$	B1	Any valid method.
Vol under curve $= \pi \int y^2\,dx$	M1	Attempt at integrating y^2
Integral $= \frac{x^3}{3} - \frac{16}{x} + 8x$	A2, 1, 0	Allow if no π present.
Uses his limits "1 to 4"	DM1	Using his limits.
$\rightarrow 75\pi - 57\pi = 18\pi$	A1 [6]	co.

Q4.

3 **(i)** Correct shape – touching positive x-axis	B1 [1]	Ignore intersections with axes
(ii) $(\pi)\int (x-2)^4\,dx$	M1	Use $(\pi)\int y^2 dx$ & attempt integrate but expansion before integn needs 5 terms
$(\pi)\left[\frac{(x-2)^5}{5}\right]$	A1	
$(\pi)[0 - (-32)/5)]$	M1	Use of limits 0, 2 on *their* $(\pi)\int y^2 dx$
$\frac{32\pi}{5}$ or 6.4π	A1 [4]	cao Rotation about y-axis max 1/5

Q5.

7 **(i)** $\frac{3(1+2x)^{-1}}{-1} + (c)$	B1	
$y = \frac{3(1+2x)^{-1}}{-2} + (c)$	B1(indep)	Division by 2 $y =$ necessary
Sub (1, (1/2))	M1	Dependent on c present
$\frac{1}{2} = \frac{3}{-6} + c \Rightarrow c = 1$	A1 [4]	Use of $y = mx + c$ etc. gets 0/4
(ii) $(1+2x)^2 (>) 9$ or $4x^2 + 4x - 8 (>) 0$ **OE**	M1	
1, −2	A1	
$x > 1, x < -2$ ISW	A1 [3]	

Q6.

9 $\frac{dy}{dx} - \frac{2}{\sqrt{x}} - 1$ $P(9, 5)$		
(i) $y - 4\sqrt{x} - x(+c)$ Uses (9, 5) in an integrated expression $\rightarrow c = 2$	B1 B1 M1 A1 [4]	Ignore + c. Substitution of point after integration. co.
(ii) $\frac{dy}{dx} - 0 \rightarrow x = 4, y = 6$	M1 A1 A1 [3]	Attempt to solve $dy/dx = 0$. x correct. y correct.
(iii) $\frac{d^2y}{d^2x} - -x^{\frac{-3}{2}} \rightarrow$ –ve $\rightarrow$ Max	B1 B1√ [2]	co. √ for correct deduction.
(iv) $\frac{dy}{dx} - -\frac{1}{3}$ Perpendicular $m = 3$ $\tan\theta = 3$ Angle is $\tan^{-1}3$ $k = 3$	M1 A1 [2]	Use of $m_1m_2 = -1$ Needs $k = 3$

Q7.

11 **(i)** $x - \frac{4}{y^2} - 1$	B1 [1]	**AG** At least 1 step of working needed
(ii) $\int\left(\frac{4}{y^2} - 1\right)dy - \left[-\frac{4}{y} - y\right]$	**B1B1**	
Upper limit = 2 $\left[\left(-\frac{4}{2} - 2\right) - (-4 - 1)\right]$ 1	**B1** **M1** **A1** **[5]**	For $-\frac{4}{y}, -y$ Apply limits 1 and *their* 2 'correctly' SC B2 for $\int 2(x+1)^{-\frac{1}{2}}\,dx - 3 \rightarrow 1$
(iii) $(\pi)\int x^2\,dy - (\pi)\int\left(\frac{16}{y^4} - \frac{8}{y^2} + 1\right)dy$	**B1B1**	
$(\pi)\left[\frac{-16}{3y^3} + \frac{8}{y} + y\right]$	**B1**	
$(\pi)\left[\left(\frac{-16}{24} + 4 + 2\right) - \left(\frac{-16}{3} + 8 + 1\right)\right]$	**M1**	Apply limits 1 and *their* 2 'correctly'
$\frac{5\pi}{3}$	**A1** **[5]**	

Q8.

5 $x - \frac{8}{y^2} - 2$; at $x = 0, y = 2$ $\rightarrow x^2 - \frac{64}{y^4} - \frac{32}{y^2} + 4$ Integral of $x^2 = \frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y$ Uses limits 1 to 2 $\rightarrow 6\frac{2}{3}\pi$	B1 B1B1B1 M1 A1 [6]	co All co. Uses 1 to 2 or 2 to 1. co.

Q9.

9 $\frac{d^2y}{dx^2} = -4x$ **(i)** $\frac{dy}{dx} = -2x^2 + c$ $\frac{dy}{dx} = 0$ when $x = 2$, $\rightarrow c = 8$ $y = -\frac{2x^3}{3} + 8x$ $(+C)$ Subs (2, 12) $\rightarrow C = \frac{4}{3}$	B1 B1 B1 B1√ M1 A1 [6]	For $-2x^2$ $c = 8$ For each term – √ on "c"– ignore $(+C)$ Uses (2, 12) to find C.
(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= -10 \times 0.05$ $\rightarrow$ decreasing at 0.5 units per second	M1 A1 [2]	 Must use. Enough to see product of gradient and rate. bod over notation.

Q10.

10 (i) $\frac{dy}{dx} = 4(x-2)^3$	**B1**	Or $4x^3 - 24x^2 + 48x - 32$
Grad of tangent = –4	**M1**	Sub $x = 1$ into *their* derivative
Eq. of tangent is $y - 1 = -4(x - 1)$	**M1**	Line thru (1, 1) and with m from deriv
$\rightarrow B(\frac{5}{4}, 0)$	**A1**	
Grad of normal $= \frac{1}{4}$	**M1**	Use of $m_1m_2 = -1$
Eq. of normal is $y - 1 = \frac{1}{4}(x-1) \rightarrow C(0, \frac{3}{4})$	**A1** **[6]**	

(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$	**M1**	
$\frac{\sqrt{17}}{4}$	**A1** **[2]**	Allow $\sqrt{\frac{17}{16}}$
(iii) $\int(x-2)^4 dx = \frac{(x-2)^5}{5}$	**B1**	Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$
$\left[0-(-\frac{1}{5})\right] = \frac{1}{5}$	**M1**	Apply limits $1 \to 2$ for curve
$\Delta = \frac{1}{2} \times 1 \times (their\ \frac{5}{4} - 1) = \frac{1}{8}$	**M1**	Or $\int_1^{\frac{5}{4}}(-4x+5)\,dx = \frac{1}{8}$
$\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ or 0.075	**A1** **[4]**	

Q11.

1	$\frac{dy}{dx} = \sqrt{2x+5}$			
	$\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$	B1 B1		B1 Everything without "÷2". B1 "÷2"
	Uses (2, 5) $\rightarrow c = -4$	M1 A1	[4]	Uses point in an integral.

Q12.

11	$y = \frac{8}{\sqrt{x}} - x$			
(i)	$\frac{dy}{dx} = -4x^{-\frac{3}{2}} - 1$	B1		needs both
	$= -\frac{3}{2}$ when $x = 4$ Eqn of BC $y - 0 = -\frac{3}{2}(x-4)$ $\rightarrow C\ (1, 4½)$	M1 M1 A1	[4]	Subs $x = 4$ into dy/dx Must be using differential + correct form of line at $B(4,0)$.
(ii)	area under curve = $\int(\frac{8}{\sqrt{x}} - x)$			
	$= \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - ½x^2$	B1 B1		(both unsimplified)
	Limits 1 to 4 $\rightarrow$ 8½	M1		Using correct limits.
	Area under tangent = ½ × 4½ × 3 = 6¾	M1		Or could use calculus)
	Shaded area = 1¾	A1	[5]	

Q13.

6	$\frac{dy}{dx} = k - 2x$		
	(i) At $x = 2, m = (k - 4)$ $x = 3$	M1	Obtains either gradient as f(k).
	$m = (k - 6)$		
	$(k - 4)(k - 6) = -1$	M1	Uses $m_1m_2 = -1$ with gradients f(k)
	$\rightarrow k = 5$	DM1A1 [4]	Soln of quadratic = 0. co (watch for fortuitous answers)
	(ii) $y = kx - x^2 (+ c)$	B1√	For integration without c
	Substitutes (4, 9)	M1	Realises need to substitute for x and y
	$\rightarrow c = 5$	A1 [3]	co (nb If $k = 5$ is fortuitous, loses last A1)

Q14.

1	$\int\left(x+\frac{1}{x}\right)^2 dx$			
	$= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$	B1 × 3	[3]	co. Omission of middle term of expansion can still get 2/3.

Q15.

11	$y = \frac{9}{2-x}$			
	(i) $\frac{dy}{dx} = -9(2-x)^{-2} \times -1$	B1 B1		Without the "× −1" Indep. With the "×−1". Indep.
	$\frac{9}{(2-x)^2} \neq 0$. No turning points.	B1√	[3]	√ provided of form $k \div (2-x)^2$.
	(ii) $V = \pi\int\frac{81}{(2-x)^2}dx$			
	$\int y^2 dx = -81(2-x)^{-1} \div (-1)$	B1 B1		Answer without the "÷ −1 including π For "÷ −1".
	Use of limits 0 to 1	M1		Uses both limits in an integral of y^2 – if "0" ignored, M0.
	$\rightarrow \frac{81\pi}{2}$ (or 127)	A1	[4]	co (If π omitted – max 3/4)
	(iii) $\frac{9}{2-x} = x + k$	M1		Elimination of y
	$\rightarrow x^2 - 2x + kx - 2k + 9 = 0$			
	Uses $b^2 - 4ac$	M1		Uses discriminant
	$\rightarrow k^2 + 4k - 32$			
	$\rightarrow$ end-points of 4 and −8	A1		End-values correct.
	Range for 2 points of intersection $\rightarrow k < -8$, $k > 4$.	A1	[4]	Accept ≤, ≥.

Q16.

11	(i)	$9 - x^3 - \dfrac{8}{x^3}$	M1		Together with attempt to mult by x^3
		$x^6 - 9x^3 + 8 = 0$	A1		**AG** completely correct working
		$(X-1)(X-8) = 0 \rightarrow X = 1$ or 8	M1		Attempt to solve quadratic in X or x^3
		$a = 1,\ b = 2$	A1	[4]	
	(ii)	$\int_1^2 \left[(9 - x^3) - \dfrac{8}{x^3}\right] dx$	M1		Intention to integrate the difference $y_1 - y_2$ not $\pi(y_1 - y_2)$
		$\left[9x - \dfrac{x^4}{4}\right] \cdot \left[\dfrac{-4}{x^2}\right]$	B1 B1		
		$18 - 4 + 1 - (9 - \dfrac{1}{4} + 4)$	M1		Correct use of their limits once
		$2\dfrac{1}{4}$	A1	[5]	
	(iii)	$\dfrac{dy}{dx} = \dfrac{-24}{x^4},\ \dfrac{dy}{dx} = -3x^2$	B1, B1		cao
		$\dfrac{-24}{c^4} = -3c^2$			
		$c^6 = 8$	M1		Equating and solution
		$c = \sqrt{2}$ or $8^{1/6}$ or $1.41(4...)$	A1	[4]	Accept x or c

Q17.

10	(i)	B $= (0,1)$ C $= (4,3)$	B1, B1	[2]	If B0B0 then SCB1 for both $y = 1$ & $x = 4$
	(ii)	$\dfrac{\delta y}{\delta x} = \dfrac{1}{2} \times 2(1+2x)^{-\frac{1}{2}}$	M1A1		$-\frac{1}{2}$ required & at least one of $\dfrac{1}{2} \times 2$ for M1
		Grad. of normal $= -3$	B1		
		$y - 3 = -3(x-4)$ or $y = -3x + 15$ oe	B1√	[4]	Ft only from *their* C
	(iii)	$y^2 = 1 + 2x \Rightarrow x = \dfrac{1}{2(y^2 - 1)}$ SOI	B1		$\int x^2 \delta y$, square $\dfrac{1}{2}(y^2 - 1)$ & attempt int"
		$(\pi) \times \dfrac{1}{4} \times \int (y^4 - 2y^2 + 1) \delta y$	M1		
		$(\pi) \times \dfrac{1}{4}\left[\dfrac{y^5}{5} - \dfrac{2y^3}{3} + y\right]$	A1		Apply limits $0 \rightarrow$ *their* 1 (from *their* B)
		$(\pi) \times \dfrac{1}{4}\left[\dfrac{1}{5} - \dfrac{2}{3} + 1\right]$	DM1		cao SCB1 for $\int y^2 \delta x \rightarrow \dfrac{\pi}{4}$ (scores 1/5)
		$\dfrac{2}{15}\pi$	A1	[5]	
			B1, B1		

Q18.

10 **(i)** $\int (x+1)^{\frac{1}{2}} - (x+1)$ or $\int (y^2-1)-(y-1)$	M1	Dealing with line as a triangle or integral with correct limits.
$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}x^2 - x$ or $\frac{1}{3}y^3 - \frac{1}{2}y^2$	M1A1	Attempt at integral of curve.
$\frac{2}{3} - \left(0 - \frac{1}{2} + 1\right)$ or $\frac{1}{3} - \frac{1}{2}$	DM1	Applying limits $-1 \to 0$ or $0 \to 1$ to curve
$\frac{1}{6}$	A1 [5]	π included loses last mark.
(ii) $V_1 = (\pi)\int (y^2-1)^2 = (\pi)\int y^4 - 2y^2 + 1$	M1	
$(\pi)\left[\frac{y^5}{5} - \frac{2y^3}{3} + y\right]$	A1	Attempt at $\int x^2$ dy for curve
$(\pi)\left[\frac{1}{5} - \frac{2}{3} + 1\right]$	DM1	Apply limits $0 \to 1$
$V_1 = \frac{8}{15(\pi)}$ or $0.533(\pi)$ (AWRT)	A1	
or $(\pi)\left[y\uparrow 3/3 - y\uparrow 2 + y\right]$	M1	
$V_2 = \frac{1}{3}\pi$	A1	Vol of cone or attempt to $\int x^2 dy$ for line
Volume $= \frac{8}{15}\pi \quad \frac{1}{-3}\pi = \frac{1}{5}\pi$ (or 0.628)	A1 [7]	
OR $(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)$	M1	
$(\pi)\int y^4 - 3y^2 + 2y$	M1	
$(\pi)\left[y\uparrow 5/5 - y\uparrow 3 + y\uparrow 2\right]$	A1,A1,A1	Attempt to $\int x^2 dy$
$(\pi)\left[\frac{1}{5} - 1 + 1\right]$	DM1	Attempt to $\int (x_1^2 - x_2^2)$
$\frac{1}{5}\pi$	A1	Apply limits $0 \to 1$ dependent on first M1

Q19.

2 $y = \frac{4}{x^2} - x \;(+c)$ Sub (2, 4) $\to c = 5$	**M1A1** **DM1A1** **[4]**	Attempt integration. cao Dependent on c present

Q20.

8 **(i)** $y^2 = 3y \Rightarrow y(y-3) = 0 \Rightarrow y = 3$ (or 0)	**M1**	**OR** form equation in x and attempt solution
$x = \frac{1}{2}$ or 5 $(\Rightarrow a = 5)$ **AG**	**A1** **[2]**	**OR** sub x=5 each eq (M1) → y = 3(twice) (A1) (5,3) subst only once scores 0/2
(ii) $\left[\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}}\right] [\div 2]$, $\left[\frac{2}{3} \times \frac{x^2}{2} - \frac{x}{3}\right]$	**B1B1B1**	Or Δ = ½(5 – ½)× 3
$\left[\frac{27}{3} - 0\right]$, $\left[\frac{25}{3} - \frac{5}{3} - \left(\frac{1}{12} - \frac{1}{6}\right)\right]$	**M1**	Apply limits ½ and 5 for, at least, curve
Subtract areas at some stage	**M1**	Dependent on some integration
$\frac{9}{4}$ oe	**A1** **[6]**	cao 9/4 with no working scores 0/6, but 9 – 27/4 = 9/4 scores 1/6 (M1 subtraction)

Q21.

11 **(i)** $a = 2$	B1 [1]	
(ii) $y =$	B1	
	B2, 1, 0 √	–1 for eeoo
	B1 [4]	cao
$b = \frac{2}{3}$	B2, 1, 0	–1 for eeoo
(iii) area = $\int y\,dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]$	M1	Apply limits 0→2 – ft *their a* from **(i)**
$\frac{4}{3}$	A1 [4]	cao
(iv) $\frac{d^2y}{dx^2} = 6x - 8 = 0 \Rightarrow x = \frac{4}{3}$	M1 A1	Attempt $\frac{d^2y}{dx^2}$ and set = 0
$1\frac{4}{3}$	DM1 A1 [4]	cao
When $x = \frac{4}{3}$, $\frac{dy}{dx}$ (or m) =		

Q22.

2 Attempt integration	**M1**	
$f(x) - 2(x+6)^{\frac{1}{2}} - \frac{6}{x}(+c)$	**A1A1**	Accept unsimplified terms
$2(3) - \frac{6}{3} + c - 1$	**M1**	Sub.x = 3, y = 1. c must be present
$c = -3$	**A1** **[5]**	

Q23.

10 (i) $\frac{dy}{dx} - \left[3(3-2x)^2\right] \times [-2]$	**B1B1**	OR $-54 + 72x - 24x^2$ B2,1,0
At $x - \frac{1}{2}, \frac{dy}{dx} - -24$	**M1**	
$y - 8 - -24\left(x - \frac{1}{2}\right)$	**DM1**	
$y - -24x + 20$	**A1** **[5]**	
(ii) Area under curve $= \left[\frac{(3-2x)^4}{4}\right] \times \left[-\frac{1}{2}\right]$	**B1B1**	OR $27x - 27x^2 + 12x^3 - 2x^4$ B2,1,0
$-2 - \left(-\frac{81}{8}\right)$	**M1**	Limits 0→ ½ applied to integral with intention of subtraction shown
Area under tangent $= \int(-24x + 20)$	**M1**	or area trap =½(20 + 8) × ½
$- \left\| -12x^2 + 20x \right\|$ or 7 (from trap)	**A1**	Could be implied
$\frac{9}{8}$ or 1.125	**A1** **[6]**	Dep on both M marks

Q24.

2 $f(x) = 2x^{-\frac{1}{2}} + x\ (+c)$	M1A1	Attempt integ $x^{-\frac{1}{2}}$ or $+x$ needed for M
$5 = -2 \times \frac{1}{2} + 4 + c$	M1	Sub (4, 5). c must be present
$c = 2$	A1 [4]	

Q25.

11 (i) $\frac{dy}{dx} - \left[\frac{1}{2}(x4 + 4x + 4)^{-\frac{1}{2}}\right] \times \left[4x^3 + 4\right]$	B1B1	
At $x = 0$, $\frac{dy}{dx} - \frac{1}{2} \times \frac{1}{2} \times 4 - (1)$	M1	Sub $x = 0$ and attempt eqn of line following differentiation.
Equation is $y - 2 = x$	A1 [4]	
(ii) $x + 2 = \sqrt{x^4 + 4x + 4} \Rightarrow (x+2)^2$	B1	**AG** www
$= x4 + 4x + 4$		
$x^2 - x^4 = 0$ oe	B1	
$x = 0, \pm 1$	B2,1,0 [4]	
(iii) $(\pi)\left[\frac{x^5}{5} + 2x^2 + 4x\right]$	M1A1	Attempt to integrate y^2
$(\pi)\left[0 - \left(\frac{-1}{5} + 2 - 4\right)\right]$	DM1	
$\frac{11\pi}{5}$ (6.91) oe	A1 [4]	Apply limits $-1 \to 0$