Q1.

3 (i)	State or imply indefinite integral of e^{2x} is $\frac{1}{2}e^{2x}$, or equivalent	B1
	Substitute correct limits correctly	M1
	Obtain answer $R = \frac{1}{2}e^{2p} - \frac{1}{2}$, or equivalent	A1
		[3]
(ii)	Substitute $R = 5$ and use logarithmic method to obtain an equation in $2p$	M1*
	Solve for p	M1 (dep*)
	Obtain answer $p = 1.2$ (1.1989 ...)	A1
		[3]

Q2.

7	(i)	Make relevant use of the cos(A + B) formula	M1*	
		Make relevant use of cos2A and sin2A formulae	M1*	
		Obtain a correct expression in terms of cosA and sinA	A1	
		Use $\sin^2 A = 1 - \cos^2 A$ to obtain an expression in terms of cosA	M1(dep*)	
		Obtain given answer correctly	A1	**5**
	(ii)	Replace integrand by $\frac{1}{4}\cos 3x + \frac{3}{4}\cos x$, or equivalent	B1	
		Integrate, obtaining $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$, or equivalent	B1 + B1√	
		Use limits correctly	M1	
		Obtain given anser	A1	**5**

Q3.

7	(i)	Make relevant use of the sin(*A* + *B*) formula	B1	
		Make relevant use of sin2*A* and cos2*A* formulae	M1	
		Obtain a correct expression in terms of sin *x* and cos *x*	A1	
		Use $\cos^2 x = 1 - \sin^2 x$ to obtain an expression in terms of sin *x*	M1(dep*)	
		Obtain given answer correctly	A1	**5**
	(ii)	Replace integrand by $\frac{3}{4}\sin x - \frac{1}{4}\sin 3x$, or equivalent	B1	
		Integrate, obtaining $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$, or equivalent	B1√ + B1√	
		Use limits correctly	M1	
		Obtain given answer correctly	A1	**5**

Q4.

6	(i)	State correct expression $\frac{1}{2}+\frac{1}{2}\cos 2x$, or equivalent	B1	[1]
	(ii)	Integrate an expression of the form $a+b\cos 2x$, where $ab \neq 0$, correctly	M1	
		State correct integral $\frac{1}{2}x+\frac{1}{4}\sin 2x$, or equivalent	A1	
		Use correct limits correctly	M1	
		Obtain given answer correctly	A1	[4]
	(iii)	Use identity $\sin^2 x = 1-\cos^2 x$ and attempt indefinite integration	M1	
		Obtain integral $x-\left(\frac{1}{2}x-\frac{1}{4}\sin 2x\right)$, or equivalent	A1	
		Use limits and obtain answer $\frac{1}{6}\pi-\frac{\sqrt{3}}{8}$	A1	[3]
		[Solutions that use the result of part **(ii)**, score M1A1 for integrating 1 and A1 for the final answer.]		

Q5.

3	Obtain integral $\frac{1}{2}\sin 2x - \cos x$	B1 + B1	
	Substitute limits correctly in an integral of the form $a \sin 2x + b \cos x$	M1	
	Use correct exact values, e.g. of $\cos\left(\frac{1}{6}\pi\right)$	M1	
	Obtain answer $1 - \frac{1}{4}\sqrt{3}$, or equivalent	A1	[5]

Q6.

3	(i)	Show or imply correct ordinates 1, 0.5, 0.414213 ...	B1	
		Use correct formula, or equivalent, with $h = 1$ and three ordinates	M1	
		Obtain answer 1.21 with no errors seen	A1	[3]
	(ii)	Justify the statement that the rule gives an over-estimate	B1	[1]

Q7.

2	Obtain integral $\ln(x + 2)$	B1	
	Substitute correct limits correctly	M1	
	Use law for the logarithm of a product, a quotient or a power	M1	
	Obtain given answer following full and correct working	A1	[4]

Q8.

2	(i)	State or imply correct ordinates 0.27067..., 0.20521..., 0.14936...	B1	
		Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates	M1	
		Obtain answer 0.21 with no errors seen	A1	[3]
	(ii)	Justify statement that the trapezium rule gives an over-estimate	B1	[1]

Q9.

4	(a)	Obtain integral $a \sin 2x$ with $a = \pm\left(1, 2 \text{ or } \frac{1}{2}\right)$	M1	
		Use limits and obtain $\frac{1}{2}$ (AG)	A1	[2]
	(b)	Use $\tan^2 x = \sec^2 x - 1$ and attempt to integrate both terms	M1	
		Obtain $3\tan x - 3x$	A1	
		Attempt to substitute limits, using exact values	M1	
		Obtain answer $2\sqrt{3} - \frac{\pi}{2}$	A1	[4]

Q10.

6	(a)	Rewrite integrand as $12e^x + 4e^{3x}$	B1	
		Integrate to obtain $12e^x$ …	B1	
		Integrate to obtain … $+\frac{4}{3}e^{3x}$	B1	
		Include … $+ c$	B1	[4]
	(b)	Use identity $\tan^2\theta = \sec^2\theta - 1$	B1	
		Integrate to obtain $2\tan\theta + \theta$ or equivalent	B1	
		Use limits correctly for integral of form $a\tan\theta + b\theta$	M1	
		Confirm given answer $\frac{1}{2}(8 + \pi)$	A1	[4]

Q11.

2	(i)	Show or imply correct ordinates 1, $\sqrt{2}$ or 1.414, 3	B1	
		Use correct formula, or equivalent, with $h = 1$	M1	
		Obtain 3.41	A1	[3]
	(ii)	Obtain 6 – 3.41 and hence 2.59, following their answer to **(i)** provided less than 6	B1√	
		Refer, in some form, to two line segments replacing curve and conclude with clear justification of given result that answer is an under-estimate.	B1	[2]

Q12.

4	(a)	Obtain integral form of $k\cos\frac{1}{2}x$	M1	
		Obtain correct $-2\cos\frac{1}{2}x$	A1	
		Use limits correctly to obtain 1	A1	[3]
	(b)	Rewrite integrand as $e^{-x} + 1$	B1	
		Integrate to obtain $-e^{-x}$ …	B1	
		Integrate to obtain … $+ x + c$	B1	[3]

Q13.

7	(i)	Expand to obtain $4\sin^2 x + 4\sin x\cos x + \cos^2 x$	B1	
		Use $2\sin x\cos x = \sin 2x$	B1	
		Attempt to express $\sin^2 x$ or $\cos^2 x$ (or both) in terms of $\cos 2x$	M1	
		Obtain correct $\frac{1}{2}k(1-\cos 2x)$ for their $k\sin^2 x$ or equivalent	A1√	
		Confirm given answer $\frac{5}{2}+2\sin 2x-\frac{3}{2}\cos 2x$	A1	[5]
	(ii)	Integrate to obtain form $px + q\cos 2x + r\sin 2x$	M1	
		Obtain $\frac{5}{2}x-\cos 2x-\frac{3}{4}\sin 2x$	A1	
		Substitute limits in integral of form $px + q\cos 2x + r\sin 2x$ and attempt simplification	DM1	
		Obtain $\frac{5}{8}\pi+\frac{1}{4}$ or exact equivalent	A1	[4]

Q14.

7	(i)	Replace $\tan^2 x$ by $\sec^2 x - 1$	B1	
		Express $\cos^2 x$ in the form $\pm\frac{1}{2}\pm\frac{1}{2}\cos 2x$	M1	
		Obtain given answer $\sec^2 x+\frac{1}{2}\cos 2x-\frac{1}{2}$ correctly	A1	
		Attempt integration of expression	M1	
		Obtain $\tan x+\frac{1}{4}\sin 2x-\frac{1}{2}x$	A1	
		Use limits correctly for integral involving at least $\tan x$ and $\sin 2x$	M1	
		Obtain $\frac{5}{4}-\frac{1}{8}\pi$ or exact equivalent	A1	[7]
	(ii)	State or imply volume is $\int \pi(\tan x+\cos x)^2\, dx$	B1	
		Attempt expansion and simplification	M1	
		Integrate to obtain one term of form $k\cos x$	M1	
		Obtain $\pi(\frac{5}{4}-\frac{1}{8}\pi)+\pi(2-\sqrt{2})$ or equivalent	A1	[4]

Q15.

3	(i)	Either		
		Use $\sin 2x = 2\sin x \cos x$ to convert integrand to $k \sin^2 2x$	M1	
		Use $\cos 4x = 1 - 2\sin^2 2x$	M1	
		State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent	A1	
		Or		
		Use $\cos^2 x = \frac{1-\cos 2x}{2}$ and/or $x = \frac{1-\cos 2x}{2}$ to obtain an equation in $\cos 2x$ only	M1	
		Use $\cos^2 2x = \frac{1+\cos 4x}{2}$	M1	
		State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent	A1	[3]
	(ii)	State correct integral $\frac{3}{2}x - \frac{3}{8}\sin 4x$, or equivalent	B1	
		Attempt to substitute limits, using exact values	M1	
		Obtain given answer correctly	A1	[3]

Q16.

1	Integrate and obtain term of the form $k \ln(7 - 2x)$	M1	
	State $y = -2\ln(7 - 2x)(+c)$	A1	
	Evaluate c	DM1	
	Obtain answer $y = -2\ln(7 - 2x) + 2$	A1√	[4]

Q17.

6	(a)	Obtain indefinite integral $-\frac{1}{2}\cos 2x + \sin x$	B1 + B1	
		Use limits with attempted integral	M1	
		Obtain answer 2 correctly with no errors	A1	4
	(b) (i)	Identify R with correct definite integral and attempt to integrate	M1	
		Obtain indefinite integral $\ln(x + 1)$	B1	
		Obtain answer $R = \ln(p + 1) - \ln 2$	A1	3
	(ii)	Use exponential method to solve an equation of the form $\ln x = k$	M1	
		Obtain answer $p = 13.8$	A1	2

Q18.

7	(i) Obtain derivative of the form $k\sec^2 2x$, where $k = 2$ or $k = 1$	M1	
	Obtain correct derivative $2\sec^2 2x$	A1	2
	(ii) State or imply the indefinite integral is $\frac{1}{k}\tan 2x$, where $k = 2$ or $k = 1$	M1*	
	Substitute limits correctly	M1(dep*)	
	Obtain given answer $\frac{1}{2}\sqrt{3}$	A1	
	Use $\tan^2 2x = \sec^2 2x - 1$ and attempt to integrate both terms, or equivalent	M1	
	Substitute limits in indefinite integral of the form $\frac{1}{k}\tan 2x - x$, where $k = 2$ or $k = 1$	M1	
	Obtain answer $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$, or equivalent	A1	6
	(iii) State that the integrand is equivalent to $\frac{1}{2}\sec^2 2x$	B1	
	Obtain answer $\frac{1}{4}\sqrt{3}$	B1	2

Q19.

1	State indefinite integral of the form $k\ \text{In}(2x + 1)$, where $k = \frac{1}{2}$, 1 or 2	M1	
	State correct integral $\frac{1}{2}\text{In}(2x + 1)$	A1	
	Use limits correctly, allow use of limits $x = 4$ and $x = 1$ in an incorrect form	M1	
	Obtain given answer	A1	[4]

Q20.

7	(i) Expand and use sin $2A$ formula	M1	
	Use cos $2A$ formula at least once	M1	
	Obtain any correct expression in terms of cos $2x$ and sin $2x$ only – can be implied	A1	
	Obtain given answer correctly	A1	[4]
	(ii) State indefinite integral $5x - 2\sin 2x - \frac{3}{2}\cos 2x$	B2	
	[Award B1 if one error in one term]		
	Substitute limits correctly – must be correct limits	M1	
	Obtain answer $\frac{1}{4}(5\pi - 2)$, or exact simplified equivalent	A1	[4]

Q21.

5	Integrate and state term $\ln x$	B1	
	Obtain term of the form $k\ln(2x + 1)$	M1	
	State correct term $-2\ln(2x + 1)$	A1	
	Substitute limits correctly	M1	
	Use law for the logarithm of a product, quotient or power	M1	
	Obtain given answer correctly	A1	[6]

Q22.

5	(i)	Use double angle formulae and obtain $a + b\cos 4x$	M1	
		Obtain answer $\frac{1}{2} + \frac{1}{2}\cos 4x$, or equivalent	A1	[2]
	(ii)	Integrate and obtain $\frac{1}{2}x + \frac{1}{8}\sin 4x$	A1√ + A1√	
		Substitute limits correctly	M1	
		Obtain answer $\frac{1}{16}\pi + \frac{1}{8}$, or exact equivalent	A1	[4]

Q23.

3	(i)	Show or imply correct ordinates 1, 1.15470…, 2	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and three ordinates	M1	
		Obtain answer 1.39 with no errors seen	A1	[3]
	(ii)	Make recognisable sketch of $y = \sec x$ for $0 \le x \le \frac{1}{3}\pi$	B1	
		Using a correct graph, explain that the rule gives an over-estimate	B1	[2]

Q24.

8	(a)	Integrate and obtain term $k\cos 2x$, where $k = \pm\frac{1}{2}$ or ± 1	M1	
		Obtain term $-\frac{1}{2}\cos 2x$	A1	
		Obtain term $\tan x$	B1	
		Substitute correct limits correctly	M1	
		Obtain exact answer $\frac{3}{4} + \sqrt{3}$	A1	[5]
	(b)	Integrate and obtain $\frac{1}{2}\ln x + \ln(x+1)$ or $\frac{1}{2}\ln 2x + \ln(x+1)$	B1 + B1	
		Substitute correct limits correctly	M1	
		Obtain given answer following full and correct working	A1	[4]

Q25.

3	Integrate and obtain $\frac{1}{2}e^{2x}$ term	B1	
	Obtain $2e^{x}$ term	B1	
	Obtain x	B1	
	Use limits correctly, allow use of limits $x = 1$ and $x = 0$ into an incorrect form	M1	
	Obtain given answer	A1	[5]
	S. R. Feeding limits into original integrand, 0/5		

Q26.

4	**(a)** Obtain integral of the form ke^{1-2x} with any non-zero k	M1	
	Correct integral	A1	[2]
	(b) Attempt to use double angle formula to expand cos $(3x + 3x)$	M1	
	State correct expression $\frac{1}{2} - \frac{1}{2}$ cos$6x$ or equivalent	A1	
	Integrate an expression of the form $a + b$ cos$6x$, where $ab \neq 0$, correctly	M1	
	State correct integral $\frac{1}{2}x - \frac{1}{12}$ sin$6x$, or equivalent	A1	[4]

Q27.

2	Integrate and obtain term of the form $k\ln(4x + 1)$	M1	
	State correct term $\frac{1}{2}\ln(4x+1)$	A1	
	Substitute limits correctly	M1	
	Use law for the logarithm of a quotient or a power	M1	
	Obtain given answer correctly	A1	[5]

Q28.

8	**(i)** Make relevant use of the $\cos(A + B)$ formula	M1*	
	Make relevant use of the cos $2A$ and sin $2A$ formulae	M1*	
	Obtain a correct expression in terms of cos x and sin x	A1	
	Use $\sin^2 x = 1 - \cos^2 x$ to obtain an expression in terms of cos x	M1(dep*)	
	Obtain given answer correctly	A1	[5]
	(ii) Replace integrand by $\frac{1}{2}\cos 3x + \frac{1}{2}\cos x$, or equivalent	B1	
	Integrate, obtaining $\frac{1}{6}\sin 3x + \frac{1}{2}\sin x$, or equivalent	B1 + B1√	
	Use limits correctly	M1	
	Obtain given answer	A1	[5]

Q29.

4	State at least one correct integral	B1	
	Use limits correctly to obtain an equation in e^{2k}, e^{4k}	M1	
	Carry out recognizable solution method for quadratic in e^{2k}	M1	
	Obtain $e^{2k} = 1$ and $e^{2k} = 3$	A1	
	Use logarithmic method to solve an equation of the form $e^{\lambda a} = b$, where $b > 0$	M1	
	Obtain answer $k = \frac{1}{2}\ln 3$	A1	[6]

Q30.

4	(i)	State correct expression $\frac{1}{2}+\frac{1}{2}\cos 2x$, or equivalent	B1	[1]
	(ii)	Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly	M1	
		State correct integral $\frac{1}{2}x+\frac{1}{4}\sin 2x$, or equivalent	A1	
		Obtain correct integral (for sin 2x term) of $-\frac{1}{2}\cos 2x$	B1	
		Attempt to substitute limits, using exact values	M1	
		Obtain given answer correctly	A1	[5]

Q31.

6	(a)	State or imply correct ordinates 0.125, 0.08743…, 0.21511…	B1	
		Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates	M1	
		Obtain answer 0.11 with no errors seen	A1	[3]
	(b)	Attempt to expand brackets and divide by e^{2x}	M1	
		Integrate a term of form ke^{-x} or ke^{-2x} correctly	A1√	
		Obtain 2 correct terms	A1	
		Fully correct integral $x + 4e^{-x} - 2e^{-2x} + c$	A1	[4]

Q32.

6	(a)	Obtain integral $ke^{-\frac{1}{2}x}$ with any non-zero k	M1	
		Correct integral	A1	[2]
	(b)	State indefinite integral of the form $k \ln (3x - 1)$, where $k = 2$, 6 or 3	M1	
		State correct integral $2 \ln (3x - 1)$	A1	
		Substitute limits correctly (must be a function involving a logarithm)	M1	
		Use law for the logarithm of a power or a quotient	M1	
		Obtain given answer correctly	A1	[5]

Q33.

6	(a) (i)	Attempt to divide by e^{2x} and attempt to integrate 2 terms	M1	
		Integrate a term of form ke^{-2x} correctly	A1√	
		Fully correct integral $x - 3e^{-2x}$ $(+ c)$	A1	[3]
	(ii)	State correct expression $\frac{1}{2}\cos 2x + \frac{1}{2}$ or equivalent	B1	
		Integrate an expression of the form $a + b\cos 2x$, where $ab \neq 0$, correctly	M1	
		State correct integral $\frac{3\sin 2x}{4}+\frac{3x}{2}(+c)$	A1	[3]
	(b)	State or imply correct ordinates 5.46143…, 4.78941…, 4.32808…	B1	
		Use correct formula, or equivalent, correctly with $h = 0.5$ and three ordinates	M1	
		Obtain answer 4.84 with no errors seen	A1	[3]

Q34.

1	(i)	State indefinite integral of the form $k \ln(4x-1)$, where $k = 2, 4$, or ½	M1	
		State correct integral $\frac{1}{2}\ln(4x-1)$	A1	[2]
	(ii)	Substitute limits correctly	M1	
		Use law for the logarithm of a power or a quotient	M1	
		Obtain ln 3 correctly	A1	[3]

Q35.

6	(a)		Expand brackets and use $\sin^2 x + \cos^2 x = 1$	M1	
			Obtain $1 - \sin 2x$	A1	
			Integrate and obtain term of form $\pm k\cos 2x$, where $k = \frac{1}{2}$, 1 or 2	M1	
			State correct integral $x + \frac{\cos 2x}{2} (+c)$	A1	[4]
	(b)	(i)	State or imply correct ordinates 1.4142…, 1.0823…, 1	B1	
			Use correct formula, or equivalent, correctly with $h = \frac{\pi}{8}$ and three ordinates	M1	
			Obtain answer 0.899 with no errors seen	A1	[3]
		(ii)	Make a recognisable sketch of $y = \operatorname{cosec} x$ for $0 < x \leqslant \frac{1}{2}\pi$	B1	
			Justify statement that the trapezium rule gives an over-estimate	B1	[2]

Q36.

5	(i)	Express left-hand side as a single fraction	M1	
		Use $\sin 2\theta = 2\sin\theta\cos\theta$ at some point	B1	
		Complete proof with no errors seen (AG)	A1	[3]

(ii)	(a)	State $\dfrac{2}{\sin\frac{1}{4}\pi}$ or equivalent	B1	
		Obtain $2\sqrt{2}$ or exact equivalent (dependent on first B1)	B1	[2]
	(b)	State or imply $k\sin 2\theta$ for any k	B1	
		Integrate to obtain $-\frac{3}{2}\cos 2\theta$	B1	
		Substitute both limits correctly to obtain 3	B1	[3]

Q37.

6	(a)	Integrate to obtain form $k\ln(2x-7)$	M1	
		Obtain correct $3\ln(2x-7)$	A1	
		Substitute limits correctly (dependent on first M1)	DM1	
		Use law for logarithm of a quotient or power (dependent on first M1)	DM1	
		Confirm $\ln 125$ following correct work and sufficient detail (AG)	A1	[5]
	(b)	Evaluate y at (1), 5, 9, 13, 17	M1	
		Use correct formula, or equivalent, with $h=4$ and five y-values	M1	
		Obtain 13.5	A1	[3]

Q38.

3	(a)	Integrate to obtain form $k\sin(\frac{1}{3}x+2)$ where $k \neq 4$	M1	
		Obtain $12\sin(\frac{1}{3}x+2)$ $(+c)$	A1	[2]
	(b)	State or imply correct y-values 2, $\sqrt{20}$, $\sqrt{68}$, $\sqrt{148}$	B1	
		Use correct formula, or equivalent, with $h=4$ and four y-values	M1	
		Obtain 79.2	A1	[3]

Q39.

1	State or imply correct y–values 6, 4, 0, 8, 24	B1	
	Use correct formula, or equivalent, with $h = 1$ and five y–values	M1	
	Obtain 27	A1	[3]

Q40.

3	(a)	Express integrand in the form $p\cos\theta + 2$	M1	
		State correct $2\cos\theta + 2$	A1	
		Integrate to obtain $2\sin\theta + 2\theta$ $(+c)$	A1	[3]
	(b)	Integrate to obtain form $k\ln(2x+3)$	M1	
		Obtain correct $\frac{1}{2}\ln(2x+3)$	A1	
		Apply limits correctly	DM1	
		Obtain $\frac{1}{2}\ln 15$	A1	[4]

Q41.

2	(i)	Integrate to obtain form $pe^{-x} + qe^{-3x}$ where $p \neq 1, q \neq 6$	M1	
		Obtain $-e^{-x} - 2e^{-3x}$ (allow unsimplified)	A1	
		Apply both limits to $pe^{-x} + qe^{-3x}$ (allow $p = 1, q = 6$)	M1	
		Obtain $3 - e^{-a} - 2e^{-3a}$	A1	[4]
	(ii)	State 3 following a result of the form $k + pe^{-x} + qe^{-3x}$	B1√	[1]

P3 (variant1 and 3)

Q1.

4	(i)	State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$	B1	
		Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent	M1	
		Simplify and obtain the given identity correctly	A1	[3]
	(ii)	Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$	B1	
		Substitute limits correctly in an integral of the form $a\sin 2x + b\sin 4x$	M1	
		Obtain given answer following full, correct and exact working	A1	[3]

Q2.

8	(i)	State or imply the form $\frac{A}{x+1}+\frac{B}{x+3}$ and use a relevant method to find A or B	M1	
		Obtain $A = 1$, $B = -1$	A1	[2]
	(ii)	Square the result of part **(i)** and substitute the fractions of part **(i)**	M1	
		Obtain the given answer correctly	A1	[2]
	(iii)	Integrate and obtain $-\frac{1}{x+1}-\ln(x+1)+\ln(x+3)-\frac{1}{x+3}$	B3	
		Substitute limits correctly in an integral containing at least two terms of the correct form	M1	
		Obtain given answer following full and exact working	A1	[5]

Q3.

7	(i)	Use correct $\cos(A+B)$ formula to express $\cos 3\theta$ in terms of trig functions of 2θ and θ	M1	
		Use correct trig formulae and Pythagoras to express $\cos 3\theta$ in terms of $\cos\theta$	M1	
		Obtain a correct expression in terms of $\cos\theta$ in any form	A1	
		Obtain the given identity correctly	A1	[4]
		[SR: Give M1 for using correct formulae to express RHS in terms of $\cos\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of either only $\cos 3\theta$ and $\cos\theta$, or only $\cos 2\theta$, $\sin 2\theta$, $\cos\theta$, and $\sin\theta$, and A1 for obtaining the given identity correctly.]		
	(ii)	Use identity and integrate, obtaining terms $\frac{1}{4}(\frac{1}{3}\sin 3\theta)$ and $\frac{1}{4}(3\sin\theta)$, or equivalent	B1 + B1	
		Use limits correctly in an integral of the form $k\sin 3\theta + l\sin\theta$	M1	
		Obtain answer $\frac{2}{3}-\frac{3}{8}\sqrt{3}$, or any exact equivalent	A1	[4]

Q4.

7	(i)	State or imply $dx = 2t\,dt$ or equivalent	B1	
		Express the integral in terms of x and dx	M1	
		Obtain given answer $\int_1^5 (2x-2)\ln x\,dx$, including change of limits **AG**	A1	[3]
	(ii)	Attempt integration by parts obtaining $(ax^2+bx)\ln x \pm \int (ax^2+bx)\frac{1}{x}dx$ or equivalent	M1	
		Obtain $(x^2-2x)\ln x - \int (x^2-2x)\frac{1}{x}dx$ or equivalent	A1	
		Obtain $(x^2-2x)\ln x - \frac{1}{2}x^2 + 2x$	A1	
		Use limits correctly having integrated twice	M1	
		Obtain $15\ln 5 - 4$ or exact equivalent	A1	[5]
		[Equivalent for M1 is $(2x-2)(ax\ln x + bx) - \int (ax\ln x + bx)\,2dx$]		

Q5.

10	(i)	Separate variables correctly and integrate of at least one side	M1	
		Carry out an attempt to find A and B such that $\frac{1}{N(1800-N)} \equiv \frac{A}{N} + \frac{B}{1800-N}$, or equivalent	M1	
		Obtain $\frac{2}{N} + \frac{2}{1800-N}$ or equivalent	A1	
		Integrates to produce two terms involving natural logarithms	M1	
		Obtain $2\ln N - 2\ln(1800-N) = t$ or equivalent	A1	
		Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a\ln N$, $b\ln(1800)$ and ct	M1	
		Obtain $2\ln N - 2\ln(1800-N) = t - 2\ln 5$ or equivalent	A1	
		Use laws of logarithms to remove logarithms	M1	
		Obtain $N = \frac{1800e^{\frac{1}{2}t}}{5+e^{\frac{1}{2}t}}$ or equivalent	A1	[9]
	(ii)	State or imply that N approaches 1800	B1	[1]

Q6.

3	Attempt integration by parts and reach $k(1-x)e^{-\frac{1}{2}x} \pm k\int e^{-\frac{1}{2}x}\,dx$, or equivalent	M1	
	Obtain $-2(1-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$, or equivalent	A1	
	Integrate and obtain $-2(1-x)e^{-\frac{1}{2}x} + 4e^{-\frac{1}{2}x}$, or equivalent	A1	
	Use limits $x = 0$ and $x = 1$, having integrated twice	M1	
	Obtain the given answer correctly	A1	[5]

Q7.

9	(i)	State or imply $\frac{dx}{dt} = k(10-x)(20-x)$ and show $k = 0.01$	B1	[1]
	(ii)	Separate variables correctly and attempt integration of at least one side	M1	
		Carry out an attempt to find A and B such that $\frac{1}{(10-x)(20-x)} \equiv \frac{A}{10-x} + \frac{B}{20-x}$, or equivalent	M1	
		Obtain $A = \frac{1}{10}$ and $B = -\frac{1}{10}$, or equivalent	A1	
		Integrate and obtain $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x)$, or equivalent	A1√	
		Integrate and obtain term $0.01t$, or equivalent	A1	
		Evaluate a constant, or use limits $t = 0$, $x = 0$, in a solution containing terms of the form $a\ln(10-x)$, $b\ln(20-x)$ and ct	M1	
		Obtain answer in any form, e.g. $-\frac{1}{10}\ln(10-x) + \frac{1}{10}\ln(20-x) = 0.01t + \frac{1}{10}\ln 2$	A1√	
		Use laws of logarithms to correctly remove logarithms	M1	
		Rearrange and obtain $x = 20(\exp(0.1t)-1)/(2\exp(0.1t)-1)$, or equivalent	A1	[9]
	(iii)	State that x approaches 10	B1	[1]

Q8.

7	Separate variables correctly and attempt integration on at least one side	M1	
	Obtain $\frac{1}{3}y^3$ or equivalent on left-hand side	A1	
	Use integration by parts on right-hand side (as far as $axe^{3x} + \int be^{3x}\,dx$)	M1	
	Obtain or imply $2xe^{3x} + \int 2e^{3x}\,dx$ or equivalent	A1	
	Obtain $2xe^{3x} - \frac{2}{3}e^{3x}$	A1	
	Substitute $x = 0$, $y = 2$ in an expression containing terms Ay^3, Bxe^{3x}, Ce^{3x}, where $ABC \neq 0$, and find the value of c	M1	
	Obtain $\frac{1}{3}y^3 = 2xe^{3x} - \frac{2}{3}e^{3x} + \frac{10}{3}$ or equivalent	A1	
	Substitute $x = 0.5$ to obtain $y = 2.44$	A1	[8]

Q9.

5	Separate variables correctly	B1	
	Integrate and obtain term $\ln x$	B1	
	Integrate and obtain term $\frac{1}{2}\ln(y^2+4)$	B1	
	Evaluate a constant or use limits $y = 0$, $x = 1$ in a solution containing $a\ln x$ and $b\ln(y^2+4)$	M1	
	Obtain correct solution in any form, e.g. $\frac{1}{2}\ln(y^2+4) = \ln x + \frac{1}{2}\ln 4$	A1	
	Rearrange as $y^2 = 4(x^2-1)$, or equivalent	A1	[6]

Q10.

4	Separate variables correctly	B1	
	Obtain term $k\ln(4-x^2)$, or terms $k_1\ln(2-x) + k_2\ln(2+x)$	B1	
	Obtain term $-2\ln(4-x^2)$, or $-2\ln(2-x) - 2\ln(2+x)$, or equivalent	B1	
	Obtain term t, or equivalent	B1	
	Evaluate a constant or use limits $x = 1$, $t = 0$ in a solution containing terms $a\ln(4-x^2)$ and bt or terms $c\ln(2-x)$, $d\ln(2+x)$ and bt	M1	
	Obtain correct solution in any form, e.g. $-2\ln(4-x^2) = t - 2\ln 3$	A1	
	Rearrange and obtain $x^2 = 4 - 3\exp(-\frac{1}{2}t)$, or equivalent (allow use of $2\ln 3 = 2.20$)	A1	[7]

Q11.

9	State or imply form $A+\frac{B}{2x+1}+\frac{C}{x+2}$	B1	
	State or obtain $A = 2$	B1	
	Use correct method for finding B or C	M1	
	Obtain $B = 1$	A1	
	Obtain $C = -3$	A1	
	Obtain $2x+\frac{1}{2}\ln(2x+1)-3\ln(x+2)$ [Deduct B1√ for each error or omission]	B3√	
	Substitute limits in expression containing $a\ln(2x+1)+b\ln(x+2)$	M1	
	Show full and exact working to confirm that $8+\frac{1}{2}\ln 9-3\ln 6+3\ln 2$, or an equivalent expression, simplifies to given result $8-\ln 9$	A1	[10]

[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in (**i**); B0√B1√B1√M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{M}{2x+1}+\frac{Nx}{x+2}$ or $\frac{Px}{2x+1}+\frac{Q}{x+2}$, give B0B0M1A0A0 in (**i**); B1√B1√B1√, if recover correct form, M1A0 in (**ii**).]

[SR: For a solution starting with $\frac{B}{2x+1}+\frac{Dx+E}{x+2}$, give M1A1 for one of $B = 1$, $D = 2$, $E = 1$ and A1 for the other two constants; then give B1B1 for $A = 2$, $C = -3$.]

[SR: For a solution starting with $\frac{Fx+G}{2x+1}+\frac{C}{x+2}$, give M1A1 for one of $C = -3$, $F = 4$, $G = 3$ and A1 for the other constants or constant; then give B1B1 for $A = 2$, $B = 1$.]

Q12.

5	(i)	Substitute for x, separate variables correctly and attempt integration of both sides	M1	
		Obtain term $\ln y$, or equivalent	A1	
		Obtain term e^{-3t}, or equivalent	A1	
		Evaluate a constant, or use $t = 0$, $y = 70$ as limits in a solution containing terms $a\ln y$ and be^{-3t}	M1	
		Obtain correct solution in any form, e.g. $\ln y - \ln 70 = e^{-3t} - 1$	A1	
		Rearrange and obtain $y = 70\exp(e^{-3t} - 1)$, or equivalent	A1	[6]
	(ii)	Using answer to part (**i**), either express p in terms of t or use $e^{-3t} \to 0$ to find the limiting value of y	M1	
		Obtain answer $\frac{100}{e}$ from correct exact work	A1	[2]

Q13.

8 **(i)**	State or imply the form $A+\frac{B}{x+1}+\frac{C}{2x-3}$	B1	
	State or obtain $A=2$	B1	
	Use a correct method for finding a constant	M1	
	Obtain $B=-2$	A1	
	Obtain $C=-1$	A1	[5]
(ii)	Obtain integral $2x-2\ln(x+1)-\frac{1}{2}\ln(2x-3)$	B3√	
	(Deduct B1√ for each error or omission. The f.t. is on A, B, C.)		
	Substitute limits correctly in an expression containing terms $a\ln(x+1)$ and $b\ln(2x-3)$	M1	
	Obtain the given answer following full and exact working	A1	[5]
	[SR: If A omitted from the form of fractions, give B0B0M1A0A0 in **(i)**; B1√B1√M1A0 in **(ii)**.]		
	[SR: For a solution starting with $\frac{B}{x+1}+\frac{Dx+E}{2x-3}$, give M1A1 for one of $B=-2$, $D=4$, $E=-7$ and A1 for the other two constants; then give B1B1 for $A=2$, $C=-1$.]		
	[SR: For a solution starting with $\frac{Fx+G}{x+1}+\frac{C}{2x-3}$ or with $\frac{Fx}{x+1}+\frac{C}{2x-3}$, give M1A1 for one of $C=-1$, $F=2$, $G=0$ and A1 for the other constants or constant; then give B1B1 for $A=2$, $B=-2$.]		

Q14.

8 **(a)**	Carry out integration by parts and reach $ax^2\ln x+b\int\frac{1}{x}x^2\,dx$	M1*	
	Obtain $2x^2\ln x-\int\frac{1}{x}.2x^2\,dx$	A1	
	Obtain $2x^2\ln x-x^2$	A1	
	Use limits, having integrated twice	M1 (dep*)	
	Confirm given result $56\ln 2-12$	A1	[5]

(b)	State or imply $\frac{du}{dx}=4\cos 4x$	B1	
	Carry out complete substitution except limits	M1	
	Obtain $\int(\frac{1}{4}-\frac{1}{4}u^2)\,du$ or equivalent	A1	
	Integrate to obtain form $k_1u+k_2u^3$ with non-zero constants k_1, k_2	M1	
	Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]

Q15.

10	(i)	State $\frac{dV}{dt} = 80 - kV$	B1	
		Correctly separate variables and attempt integration of one side	M1	
		Obtain $a\ln(80 - kV) = t$ or equivalent	M1*	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent	A1	
		Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent	A1	
		Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly	A1	[7]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain final answer 0.14	A1	
		Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
	(iii)	State a value between 530 and 540 cm^3 inclusive	B1	
		State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive)	B1	[2]

Q16.

4	(i)	State $R = 2$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
	(ii)	Substitute denominator of integrand and state integral $k\tan(x - \alpha)$	M1*	
		State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
		Substitute limits	M1 (dep*)	
		Obtain the given answer correctly	A1	[4]

Q17.

8	(i)	Separate variables correctly and integrate at least one side	M1	
		Obtain term $\ln t$, or equivalent	B1	
		Obtain term of the form $a\ln(k-x^3)$	M1	
		Obtain term $-\frac{2}{3}\ln(k-x^3)$, or equivalent	A1	
		EITHER: Evaluate a constant or use limits $t=1, x=1$ in a solution containing $a\ln t$ and $b\ln(k-x^3)$	M1*	
		Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k-x^3)+\frac{2}{3}\ln(k-1)$	A1	
		Use limits $t=4, x=2$, and solve for k	M1(dep*)	
		Obtain $k=9$	A1	
		OR: Using limits $t=1, x=1$ and $t=4, x=2$ in a solution containing $a\ln t$ and $b\ln(k-x^3)$ obtain an equation in k	M1*	
		Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k-8)+\frac{2}{3}\ln(k-1)$	A1	
		Solve for k	M1(dep*)	
		Obtain $k=9$	A1	
		Substitute $k=9$ and obtain $x=(9-8t^{-\frac{3}{2}})^{\frac{1}{3}}$	A1	[9]
	(ii)	State that x approaches $9^{\frac{1}{3}}$, or equivalent	B1√	[1]

Q18.

5	(i)	*EITHER*: Use double angle formulae correctly to express LHS in terms of trig functions of 2θ	M1	
		Use trig formulae correctly to express LHS in terms of $\sin\theta$, converting at least two terms	M1	
		Obtain expression in any correct form in terms of $\sin\theta$	A1	
		Obtain given answer correctly	A1	
		OR: Use double angle formulae correctly to express RHS in terms of trig functions of 2θ	M1	
		Use trig formulae correctly to express RHS in terms of $\cos 4\theta$ and $\cos 2\theta$	M1	
		Obtain expression in any correct form in terms of $\cos 4\theta$ and $\cos 2\theta$	A1	
		Obtain given answer correctly	A1	[4]
	(ii)	State indefinite integral $\frac{1}{4}\sin 4\theta - \frac{4}{2}\sin 2\theta + 3\theta$, or equivalent	B2	
		(award B1 if there is just one incorrect term)		
		Use limits correctly, having attempted to use the identity	M1	
		Obtain answer $\frac{1}{32}(2\pi-\sqrt{3})$, or any simplified exact equivalent	A1	[4]

Q19.

10	**(i)**	State or imply $\frac{dA}{dt} = kV$	M1*	
		Obtain equation in r and $\frac{dr}{dt}$, e.g. $8\pi r \frac{dr}{dt} = k\frac{4}{3}\pi r^3$	A1	
		Use $\frac{dr}{dt} = 2$, $r = 5$ to evaluate k	M1(dep*)	
		Obtain given answer	A1	[4]
	(ii)	Separate variables correctly and integrate both sides	M1	
		Obtain terms $-\frac{1}{r}$ and $0.08t$, or equivalent	A1 + A1	
		Evaluate a constant or use limits $t = 0$, $r = 5$ with a solution containing terms of the form $\frac{a}{r}$ and bt	M1	
		Obtain solution $r = \frac{5}{(1-0.4t)}$, or equivalent	A1	[5]
	(iii)	State the set of values $0 \leqslant t < 2.5$, or equivalent [Allow $t < 2.5$ and $0 < t < 2.5$ to earn B1.]	B1	[1]

Q20.

5	**(i)**	State or imply $dx = 2\cos\theta\, d\theta$, or $\frac{dx}{d\theta} = 2\cos\theta$, or equivalent	B1	
		Substitute for x and dx throughout the integral	M1	
		Obtain the given answer correctly, having changed limits and shown sufficient working	A1	[3]
	(ii)	Replace integrand by $2 - 2\cos 2\theta$, or equivalent	B1	
		Obtain integral $2\theta - \sin 2\theta$, or equivalent	B1√	
		Substitute limits correctly in an integral of the form $a\theta \pm b\sin 2\theta$, where ab þ 0	M1	
		Obtain answer $\frac{1}{3}\pi - \frac{\sqrt{3}}{2}$ or exact equivalent	A1	[4]
		[The f.t. is on integrands of the form $a + c\cos 2\theta$, where ac þ 0.]		

Q21.

10	(i)	State or imply $\frac{dx}{dt} = k(20 - x)$	B1	
		Show that $k = 0.05$	B1	[2]
	(ii)	Separate variables correctly and integrate both sides	B1	
		Obtain term $-\ln(20 - x)$, or equivalent	B1	
		Obtain term $\frac{1}{20}t$, or equivalent	B1	
		Evaluate a constant or use limits $t = 0$, $x = 0$ in a solution containing terms $a \ln(20 - x)$ and bt	M1*	
		Obtain correct answer in any form, e.g. $\ln 20 - \ln(20 - x) = \frac{1}{20}t$	A1	[5]
	(iii)	Substitute $t = 10$ and calculate x	M1(dep*)	
		Obtain answer $x = 7.9$	A1	[2]
	(iv)	State that x approaches 20	B1	[1]

Q22.

5	State or imply form $\frac{A}{2x+1} + \frac{B}{x+2}$	B1	
	Use relevant method to find A or B	M1	
	Obtain $\frac{4}{2x+1} - \frac{1}{x+2}$	A1	
	Integrate and obtain $2\ln(2x+1) - \ln(x+2)$ (ft on their A, B)	B1√B1√	
	Apply limits to integral containing terms $a\ln(2x+1)$ and $b\ln(x+2)$ and apply a law of logarithms correctly.	M1	
	Obtain given answer $\ln 50$ correctly	A1	[7]

Q23.

9	(i)	State $\frac{dA}{dt} = k\sqrt{2A-5}$	B1	[1]
	(ii)	Separate variables correctly and attempt integration of each side	M1	
		Obtain $(2A-5)^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Obtain $= kt$ or equivalent	A1	
		Use $t = 0$ and $A = 7$ to find value of arbitrary constant	M1	
		Obtain $C = 3$ or equivalent	A1	
		Use $t = 10$ and $A = 27$ to find k	M1	
		Obtain $k = 0.4$ or equivalent	A1	
		Substitute $t = 20$ and values for C and k to find value of A	M1	
		Obtain 63	cwo A1	[9]

Q24.

4	Separate variables and attempt integration of at least one side	M1	
	Obtain term $\ln(x+1)$	A1	
	Obtain term $k \ln \sin 2\theta$, where $k = \pm1, \pm2$, or $\pm\frac{1}{2}$	M1	
	Obtain correct term $\frac{1}{2}\ln \sin 2\theta$	A1	
	Evaluate a constant, or use limits $\theta = \frac{1}{12}\pi$, $x = 0$ in a solution containing terms $a \ln(x+1)$ and $b \ln \sin 2\theta$	M1	
	Obtain solution in any form, e.g. $\ln(x+1) = \frac{1}{2}\ln \sin 2\theta - \frac{1}{2}\ln\frac{1}{2}$ (f.t. on $k = \pm1, \pm2$, or $\pm\frac{1}{2}$)	A1√	
	Rearrange and obtain $x = \sqrt{(2\sin 2\theta)} - 1$, or simple equivalent	A1	[7]

Q25.

8	(i)	Use any relevant method to determine a constant	M1	
		Obtain one of the values $A = 3$, $B = 4$, $C = 0$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[4]
	(ii)	Integrate and obtain term $-3\ln(2-x)$	B1√	
		Integrate and obtain term $k\ln(4+x^2)$	M1	
		Obtain term $2\ln(4+x^2)$	A1√	
		Substitute correct limits correctly in a complete integral of the form $a\ln(2-x) + b\ln(4+x^2)$, $ab \neq 0$	M1	
		Obtain given answer following full and correct working	A1	[5]

Q26.

4	(i)	Separate variables and attempt integration on both sides	M1*	
		Obtain $2N^{0.5}$ on left-hand side or equivalent	A1	
		Obtain $-60e^{-0.02t}$ on right-hand side or equivalent	A1	
		Use 0 and 100 to evaluate a constant or as limits in a solution containing terms $aN^{0.5}$ and $be^{-0.02t}$	DM1*	
		Obtain $2N^{0.5} = -60e^{-0.02t} + 80$ or equivalent	A1	
		Conclude with $N = (40 - 30e^{-0.02t})^2$ or equivalent	A1	[6]
	(ii)	State number approaches 1600 or equivalent, following expression of form $(c + de^{-0.02t})^n$	B1√	[1]

Q27.

10	**(i)**	State or imply $\frac{du}{dx} = \sec^2 x$	B1	
		Express integrand in terms of u and du	M1	
		Integrate to obtain $\frac{u^{n+1}}{n+1}$ or equivalent	A1	
		Substitute correct limits correctly to confirm given result $\frac{1}{n+1}$	A1	[4]
	(ii)	**(a)** Use $\sec^2 x = 1 + \tan^2 x$ twice	M1	
		Obtain integrand $\tan^4 x + \tan^2 x$	A1	
		Apply result from part **(i)** to obtain $\frac{1}{3}$	A1	[3]
		Or Use $\sec^2 x = 1 + \tan^2 x$ and the substitution from **(i)**	M1	
		Obtain $\int u^2 du$	A1	
		Apply limits correctly and obtain $\frac{1}{3}$	A1	
		(b) Arrange, perhaps implied, integrand to $t^9 + t^7 + 4(t^7 + t^5) + t^5 + t^3$	B1	
		Attempt application of result from part **(i)** at least twice	M1	
		Obtain $\frac{1}{8} + \frac{4}{6} + \frac{1}{4}$ and hence $\frac{25}{24}$ or exact equivalent	A1	[3]

Q28.

6	Separate variables correctly and attempt integration of one side	B1	
	Obtain term $\ln x$	B1	
	State or imply and use a relevant method to find A or B	M1	
	Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$		
	Integrate and obtain $-\frac{1}{2}\ln(1-y) + \frac{1}{2}\ln(1+y)$, or equivalent	A1 √	
	[If the integral is directly stated as $k_1 \ln$ or $k_2 \ln$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$]		
	Evaluate a constant, or use limits $x = 2$, $y = 0$ in a solution containing terms $a \ln x$, $b \ln(1-y)$ and $c \ln(1+y)$, where $abc \neq 0$	M1	
	[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k\ln(1-y^2)$]		
	Obtain solution in any correct form, e.g. $\frac{1}{2}\ln \;= \ln x - \ln 2$	A1	
	Rearrange and obtain $y =$, or equivalent, free of logarithms	A1	[8]

Q29.

4	Separate variables correctly and integrate one side	M1	
	Obtain $\ln y = \ldots$ or equivalent	A1	
	Obtain $= 3\ln(x^2+4)$ or equivalent	A1	
	Evaluate a constant or use $x = 0$, $y = 32$ as limits in a solution containing terms $a \ln y$ and $b\ln\left(x^2+4\right)$	M1	
	Obtain $\ln y = 3\ln(x^2+4) + \ln 32 - 3\ln 4$ or equivalent	A1	
	Obtain $y = \frac{1}{2}\left(x^2+4\right)$ or equivalent	A1	[6]

Q30.

7 (i)	State or imply $du = 2\cos 2x \, dx$ or equivalent	B1	
	Express integrand in terms of u and du	M1	
	Obtain $\int \frac{1}{2} u^3\left(1-u^2\right) du$ or equivalent	A1	
	Integration to obtain an integral of the form $k_1 u^4 + k_2 u^6, k_1, k_2 \neq 0$	M1	
	Use limits 0 and 1 or (if reverting to x) 0 and $\frac{1}{4}\pi$ correctly	DM1	
	Obtain $\frac{1}{24}$, or equivalent	A1	[6]
(ii)	Use 40 and upper limit from part **(i)** in appropriate calculation	M1	
	Obtain $k = 10$ with no errors seen	A1	[2]

Q31.

3	*EITHER:* Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$	M1*	
	Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent	A1	
	Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4(\ln 4 - 1)$, or exact equivalent	A1	
	OR1: Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u} - m\int e^{\frac{1}{2}u}\,du$	M1*	
	Obtain $2ue^{\frac{1}{2}u} - 2\int e^{\frac{1}{2}u}\,du$, or equivalent	A1	
	Integrate again and obtain $2ue^{\frac{1}{2}u} - 4e^{\frac{1}{2}u}$, or equivalent	A1	
	Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	
	OR2: Using $u = \sqrt{x}$, or equivalent, integrate and obtain $ku\ln u - m\int u.\frac{1}{u}\,du$	M1*	
	Obtain $4u\ln u - 4\int 1\,du$, or equivalent	A1	
	Integrate again and obtain $4u\ln u - 4u$, or equivalent	A1	
	Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int \ln u\,du$ as $u\ln u \pm u$	M1(dep*)	
	Obtain answer $8\ln 2 - 4$, or exact equivalent	A1	
	OR3: Integrate by parts and reach $I = \frac{x\ln x \pm x}{\sqrt{x}} + k\int \frac{x\ln x \pm x}{x\sqrt{x}}\,dx$	M1*	
	Obtain $I = \frac{x\ln x - x}{\sqrt{x}} + \frac{1}{2}I - \frac{1}{2}\int \frac{1}{\sqrt{x}}\,dx$	A1	
	Integrate and obtain $I = 2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
	Obtain answer $4\ln 4 - 4$, or exact equivalent	A1	[5]

Q32.

5	**(i)**	Use Pythagoras	M1	
		Use the $\sin 2A$ formula	M1	
		Obtain the given result	A1	[3]
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	[4]

Q33.

10	(i) State or imply $V = \pi h^3$	B1	
	State or imply $\frac{dV}{dt} = -k\sqrt{h}$	B1	
	Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$, or equivalent	M1	
	Obtain the given equation	A1	[4]
	[The M1 is only available if $\frac{dV}{dh}$ is in terms of h and has been obtained by a correct method.]		
	[Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.]		
	(ii) Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60, h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
	(ii) Obtain final answer $t = 60\left(1-\left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	[6]
	(iii) Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	[1]

Q34.

2	Carry out complete substitution including the use of $\frac{du}{dx} = 3$	M1	
	Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$	A1	
	Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$	M1	
	Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c	A1	[4]

Q35.

10	Use $2\cos^2 x = 1 + \cos 2x$ or equivalent	B1
	Separate variables and integrate at least one side	M1
	Obtain $\ln(y^3 + 1) = \ldots$ or equivalent	A1
	Obtain $\ldots = 2x + \sin 2x$ or equivalent	A1
	Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln(y^3 + 1)$, bx or $c\sin 2x$	M1*
	Obtain $\ln(y^3 + 1) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant	A1
	Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point	B1
	Use correct process to find y-coordinate for at least one x-coordinate	M1(d*M)
	Obtain 5.9	A1
	Obtain 48.1	A1 [10]

Q36.

2	State $\frac{du}{dx} = 3\sec^2 x$ or equivalent	B1	
	Express integral in terms of u and du (accept unsimplified and without limits)	M1	
	Obtain $\int \frac{1}{3} u^{\frac{1}{2}}\, du$	A1	
	Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3} u^{\frac{3}{2}}$	M1	
	Obtain $\frac{14}{9}$	A1	[5]

Q37.

4	Separate variables correctly and recognisable attempt at integration of at least one side	M1	
	Obtain $\ln y$, or equivalent	B1	
	Obtain $k\ln(2 + e^{3x})$	B1	
	Use $y(0) = 36$ to find constant in $y = A(2 + e^{3x})^k$ or $\ln y = k\ln(2 + e^{3x}) + c$ or equivalent	M1*	
	Obtain equation correctly without logarithms from $\ln y = \ln\left(A(2 + e^{3x})^k\right)$	*M1	
	Obtain $y = 4(2 + e^{3x})^2$	A1	[6]

Q38.

5	Separate variables correctly and attempt integration of at least one side	B1	
	Obtain term in the form $a\sqrt{(2x+1)}$	M1	
	Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
	Obtain term of the form $k\tan\theta$	M1	
	Evaluate a constant, or use limits $x=0, \theta=\frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k\tan\theta$, $ak\neq 0$	M1	
	Obtain correct solution in any form, e.g. $\sqrt{(2x+1)}=\frac{1}{2}\tan\theta+\frac{1}{2}$	A1	
	Rearrange and obtain $x=\frac{1}{8}(\tan\theta+1)^2-\frac{1}{2}$, or equivalent	A1	7

Q39.

8	(i)	Use a correct method for finding a constant	M1	
		Obtain one of $A=3$, $B=3$, $C=0$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	4
	(ii)	Integrate and obtain term $-3\ln(2-x)$	B1√	
		Integrate and obtain term of the form $k\ln(2+x^2)$	M1	
		Obtain term $\frac{3}{2}\ln(2+x^2)$	A1√	
		Substitute limits correctly in an integral of the form $a\ln(2-x)+b\ln(2+x^2)$, where $ab\neq 0$	M1	
		Obtain given answer after full and correct working	A1	5

Q40.

2	(i)	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h=\frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	[3]
	(ii)	Make recognisable sketch of $y=\operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	[2]

Q41.

7	(i)	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term $\ln R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80-0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285-0.57x)}$	A1	[6]
	(ii)	Equate $\frac{dR}{dx}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	[3]

Q42.

6	(i)	State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
		Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
		Obtain 0.255 with no errors seen	A1	[3]
	(ii)	Obtain or imply $a = -6$	B1	
		Obtain x^4 term including correct attempt at coefficient	M1	
		Obtain or imply $b = 27$	A1	
		<u>Either</u> Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1√	
		Obtain 0.259	B1	
		<u>Or</u> Use correct trapezium rule with at least 3 ordinates	M1	
		Obtain 0.259 (from 4)	A1	[5]

Q43.

8	(i)	Sensibly separate variables and attempt integration of at least one side	M1	
		Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\,dx$	M1	
		Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\,dx$ or equivalent	A1	
		Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
		Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
	(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
		Substitute 25 and calculate value of y	DM*1	
		Obtain 203	A1	[3]

Q44.

10	State or imply $\frac{du}{dx} = e^x$	B1	
	Substitute throughout for x and dx	M1	
	Obtain $\int \frac{u}{u^2 + 3u + 2} du$ or equivalent (ignoring limits so far)	A1	
	State or imply partial fractions of form $\frac{A}{u+2} + \frac{B}{u+1}$, following their integrand	B1	
	Carry out a correct process to find at least one constant for their integrand	M1	
	Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	A1	
	Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
	Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
	Apply appropriate limits and use at least one logarithm property correctly	M1	
	Obtain given answer $\ln\frac{8}{5}$ legitimately	A1	[10]
	SR for integrand $\frac{u^2}{u(u+1)(u+2)}$		
	State or imply partial fractions of form $\frac{A}{u} + \frac{B}{u+1} + \frac{C}{u+2}$	(B1)	
	Carry out a correct process to find at least one constant	(M1)	
	Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	(A1)	
	…complete as above.		