Q1.

5 $x \mapsto 2\sin^2 x - 3\cos^2 x$		
(i) $2(1-\cos^2 x) - 3\cos^2 x$	M1	Uses $s^2 + c^2 - 1$
$\rightarrow\ 2 - 5\cos^2 x \quad (a = 2, b = -5)$	A1 [2]	co
(ii) Values are −3 and 2	B1√ B1√ [2]	
(iii) $2 - 5\cos^2 x = -1$		
$\rightarrow \cos^2 x = 0.6$	B1√	
$x = 0.685, 2.46$ (accept 0.684)	B1 B1√ [3]	co √ for $\pi -$ (first answer) SC B1 for both 39.2 and 140.8

Q2.

9 **(i)** $2x^2 - 12x + 7 = 2(x-3)^2 - 11$	3 × B1 [3]	B1 for each value – accept if *a*, *b*, *c* not specifically quoted.
(ii) Range of f $\geqslant -11$	B1√ [1]	√ to his "*c*". allow > or ⩾.
(iii) $2x^2 - 12x + 7 < 21$		
$\rightarrow 2x^2 - 12x - 14$ or $2(x-3)^2 < 32$	M1	3-term quadratic to 0 or $2(x-3)^2 < 32$
$\rightarrow$ end-values of 7 or −1	A1	Correct end-values
$\rightarrow -1 < x < 7$	A1 [3]	co
(iv) $\text{gf}(x) = 2(2x^2 - 12x + 7) + k = 0$	M1 A1	Puts f into g. co.
Use of $b^2 - 4ac$	M1	Used correctly with quadratic
$\rightarrow 24^2 - 16(14 + k)$		
$\rightarrow k = 22$	A1 [4]	co.

Q3.

3 $\text{f}: x \mapsto a + b\cos x$		
(i) $\text{f}(0) = 10,\ a + b = 10$		
$\text{f}(^2/_3\pi) = 1,\ a - \frac{b}{2} - 1$	B1	EITHER OF THESE
$\rightarrow a = 4, b = 6$	B1 [2]	both co
(ii) Range is −2 to 10.	B1√ [1]	√ for his "*a* − *b*" to "*a* + *b*"
(iii) $\cos\left(\frac{5}{6}\pi\right) - -\cos\left(\frac{1}{6}\pi\right) - -\frac{\sqrt{3}}{2}$	B1	For $\cos 30° - \frac{1}{2}\sqrt{3}$ used somewhere.
$\rightarrow 4 - 3\sqrt{3}$	B1 [2]	co

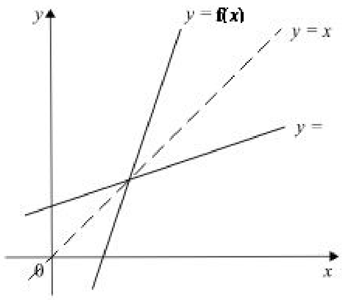
Q4.

10 f: $x \mapsto 2x^2 - 8x + 14$		
(i) $y + kx = 12$, Sim Eqns. $\rightarrow 2x^2 - 8x + kx + 2 = 0$ Use of $b^2 - 4ac$ $\rightarrow (k-8)^2 = 16 \rightarrow k = 12$ or 4.	M1 A1 M1 A1 [4]	Complete elimination of y (or x) Uses $b^2 - 4ac$ on eqn = 0, no "x" in a, b, c. co.co
(ii) $2x^2 - 8x + 14 = 2(x-2)^2 + 6$	B1×3 [3]	
(iii) Range of f $\geqslant 6$.	B1√ [1]	√ for c or from calculus.
(iv) Smallest $A = 2$	B1√ [1]	√ to answer to **(ii)**.
(v) Makes x the subject Order of operations correct. $g^{-1}(x) - \sqrt{\frac{x-6}{2}} + 2$	M1 M1 A1 [3]	Could interchange x, y first. Order must be correct. co

Q5.

11 **(i)** $fg(x) = 2x^2 - 3$, $gf(x) = 4x^2 + 4x - 1$	B1, B1 [2]	fg & gf clearly transposed gets B0B0
(ii) $2a^2 - 3 - 4a^2 + 4a - 1 \Rightarrow 2a^2 + 4a + 2 - 0$ $(a+1)^2 = 0$ $a = -1$	M1 M1 A1 [3]	Dep. quadratic. Allow x for all 3 marks Allow marks in **(ii)** if transposed in **(i)**
(iii) $b^2 - b - 2 = 0 \rightarrow (b+1)(b-2) = 0$ $b = 2$ Allow $b = -1$ in addition	M1 A1 [2]	Allow in terms of x for M1 only Correct answer without working B2
(iv) $f^{-1}(x) - \frac{1}{2}(x-1)$ $f^{-1}g(x) - \frac{1}{2}(x^2 - 3)$	B1 B1√ [2]	 Must be simplified. Ft from *their* f^{-1}
(v) $x - (\pm)\sqrt{y+2}$ $h^{-1}(x) - -\sqrt{x+2}$	M1 A1 [2]	

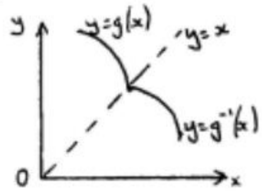
Q6.

10 $f: x \mapsto 3x-4$ $\quad g: x \mapsto 2(x-1)^3+8$		
(i) $fg(2) = f(10) = 26$ $\quad f^{-1}(x)$	M1 A1 [2]	Must use g first, then f. co
(ii) 	B1 B1 B1 [3]	$y = f(x)$ correct in 1st, 4th quadrants. $y = f^{-1}(x)$ correct in 1st, 2nd quadrants. $y = x$ marked, or quoted.
(iii) $g'(x) = 6(x-1)^2$ $g'(x) > \rightarrow$ no turning points $\rightarrow$ g is 1 : 1, g has an inverse.	B1 B1√ B1√ [3]	co allow only for incorrect "6" following from incorrect "6"
(iv) $f^{-1}(x) = \frac{x+4}{3}$ Attempt at making x Order correct. $-8, \div 2, \sqrt[3]{\ }, +1$ $g^{-1}(x) = \sqrt[3]{\frac{x-8}{2}} + 1$	B1 M1 M1 A1 [4]	co May change x and y first. Must all be correct, but allow for + 8, – 1 co as function of x, not y.

Q7.

8 **(i)** $(x-2)^2 - 4 + k$	B1B1 [2]	$a = -2, b = -4$
(ii) $f(x) \geq k-4$ or $[k-4, \infty]$ or $(k-4, \infty)$ oe	B1√ [1]	ft *their* $k-4$. Accept >
(iii) smallest value of $p = 2$	B1√ [1]	ft *their* 2
(iv) $x - 2 - (\pm)\sqrt{y+4-k}$ $x - 2 + \sqrt{y+4-k}$ $f^{-1}(x) - 2 + \sqrt{x+4-k}$ Domain is $x \geq k-4$ or $[k-4, \infty]$ or $(k-4, \infty)$ oe	M1 A1√ A1 B1√ [4]	 ft from *their* part **(i)** cao ft from *their* part **(ii)**. Accept >

Q8.

11 $f(x) = 8-(x-2)^2$,		
(i) Stationary point at $x = 2$	B1	co
y – coordinate = 8	B1	co
Nature Maximum	B1	co independent of first two marks
(or y $y - -x^2 + 4x + 4$		
$-2x + 4 = 0 \rightarrow (2, 8)$ Max)	[3]	
(ii) $k = 2$	B1√ [1]	√ on "x-value"
(iii) $y = 8-(x-2)^2$		
$\rightarrow (x-2)^2 + y - 8$	M1	Attempt to make x the subject
$\rightarrow (x-2) = \pm\sqrt{8-y}$	M1	Order of operations correct
$\rightarrow g^{-1} = 2+\sqrt{8-x}$	A1 [3]	Must be f(x).
(iv) 	B1 B1 B1 [3]	B1 arc 1st quad (no tp, no axes) B1 Evidence of symmetry about $y = x$ B1 all correct as shown left

Q9.

8 **(i)** $2(x-3)^2 - 5$ or $a = 2, b = -3, c = -5$	**B1B1B1** **[3]**	
(ii) 3	**B1** √ **[1]**	ft on – *their b*. Allow $k \geq 3$ or $x \geq 3$

(iii) $(y) \geq 27$	**B1** **[1]**	Allow >. Allow $27 \leq y \leq \infty$ etc. **OR** (x/y interchange as 1st operation)
(iv) $2(x-3)^2 - (y+5)$	**M1**	$x - 2(y-3)^2 - 5$
$x-3-(\pm)\sqrt{\frac{1}{2}(y+5)}$	**M1**	$(y-3)^2 - \frac{1}{2}(x+5)$
$x-3+/\pm\sqrt{\frac{1}{2}(y+5)}$	**A1** √	$y-3-(\pm)\sqrt{\frac{1}{2}(x+5)}$
$(f^{-1}(x))-3+\sqrt{\frac{1}{2}(x+5)}$ for $x \geq 27$	**A1B1** √ **[5]**	ft on *their* 27 from **(iii)**

Q10.

10	$f: x \mapsto 2x+k,\ g: x \mapsto x^2-6x+8,$			
(i)	$2(2x+3)+3=25$ $\to x=4$ or $\{f(11)=25,\ f(4)=11\}$	M1 A1	[2]	ff(x) needs to be correctly formed
(ii)	$x^2-6x+8=2x+k$ $x^2-8x+8-k=0$ Uses $b^2-4ac<0$ $\to k<-8$	M1 M1 A1	[3]	Eliminates y to form eqn in x. Uses the discriminant – even if =0,>0
(iii)	$x^2-6x+8=(x-3)^2-1$ $y=(x-3)^2-1$ Makes x the subject $\to \pm\sqrt{(x+1)}+3$ Needs specifically to lose the "–".	B1 B1 M1 A1√	[4]	For "–3" and "–1" Makes x the subject, in terms of x and without – or ±.

Q11.

10 $f: x \mapsto 2x+1,\ x \in \mathbb{R},\ x>0$ $g: x \mapsto \frac{2x-1}{x+3},\ x \in \mathbb{R},\ x \neq -3.$		
(i) $gf(x) = \frac{2(2x+1)-1}{2x+1+3} = \frac{4x+1}{2x+4}$ Equates to $x \to 2x^2=1$ $\to x = \frac{1}{2}\sqrt{2}$	M1 M1 A1 [3]	Must be gf, needs x replacing twice. Forms quadratic + solution Co. condone ±.
(ii) $f^{-1}(x) = \frac{1}{2}(x-1)$ To find $g^{-1}(x)$, make x the subject Order must be correct $\to g^{-1}(x) = \frac{-1-3x}{x-2}$ or $\frac{1+3x}{2-x}$	B1 M1 M1 A1 [4]	Co Attempt at x as the subject. Order correct. Allow for sign errors. Co – must be f(x).
(iii) $\frac{1+3x}{2-x} = x \to x^2+x+1=0$ Looks at b^2-4ac $\to$ negative $\to$ no roots.	M1 M1 A1 [3]	Forms quadratic equation. Looks at discriminant or attempts to solve and finds √(negative). Co
(iv)	B1 B1 B1 [3]	Correct $y=2x+1$ on graph from (0, 1) Correct $y=\frac{1}{2}(x-1)$ on graph from (1, 0) (if $-$ve x plotted, B1 s.c. for both) Shows or states or implies that f, f^{-1} are reflections in $y=x$.

Q12.

3	$f: x \mapsto 2x+3,$ $g: x \mapsto x^2-2x,$ $gf(x) = (2x+3)^2 - 2(2x+3)$ $= 4x^2+8x+3$ $= 4(x+1)^2-1$	 M1 A1 3 × B1√	[5]	 Must be f into g, not g into f. co Allow all these as √ for either fg or gf.

Q13.

6	(i) $(3x+5)(x-1)(>0)$	M1		Attempt at factorisation
	$-5/3,\ 1$	A1		Both required
	$x<-5/3,\ x>1$	A1		Ignore any words between answers
			[3]	Condone ≤ ≥
	(ii) $f(x)=x^3+x^2-5x\ (+c)$	M1		Attempt at integration
		A1		Any unsimplified expression ok
	$3=1+1-5+c$	M1		Sub. (1, 3)
	$f(x)=x^3+x^2-5x+6$	A1	[4]	Accept $c=6$

Q14.

7	(i) Range is $0\le f(x)\le 4$, 0 to 4	B1	[1]	Accept in two parts. Condone <
	(ii) $y=x$ drawn or implied	B1		
	Correct sketch of f^{-1}	B1	[2]	SC if f missing, (2, 2) (4, 6) must be shown
	(iii) $(x\mapsto)\sqrt{2x}$ for $0\le x\le 2$	B1B1		Condone < ≤
	$(x\mapsto)2x-2$ for $2<x\le 4$	B1B1	[4]	

Q15.

11	(i) $2(x-2)^2+2$	B1, B1, B1	[3]	For $2,\ -2,\ 2$
	(ii) $2\le f(x)\le 10$ oe	B1	[1]	Allow < etc. Ignore notation
	(iii) $2\le x\le 10$	B1√	[1]	Ft from part (ii). Ignore notation
	(iv) $f(x)$: ≈ half parabola from $(0,10)$ to $(2,2)$	B1		Or from int with y axis to int with *their* $y=x$
	$g(x)$: line through 0 at ≈ 45°	B1		
	$f^{-1}(x)$: reflection of *their* $f(x)$ in $g(x)$	B1√		
	Everything totally correct	B1	[4]	

	(v) $(x-2)^2=\frac{1}{2}(y-2)$	M1		Allow $+\sqrt{}$ or $-\sqrt{}$. Dep on final ans as f^n of x
	$x=2\pm\sqrt{\frac{1}{2}(y-2)}$	M1		
	$f^{-1}(x)=2-\sqrt{\frac{1}{2}(x-2)}$	A1	[3]	cao

Q16.

9 **(i)** $f^{-1}(x) - \frac{1}{2}x - \frac{3}{2}$ $2x + 3 - \frac{1}{2}x - \frac{3}{2} \Rightarrow x - -3$	B1 M1A1 [3]	
(ii) 2 lines approximately correct, reflected in $y=x$ & meeting at $(-3, -3)$	B3,2,1,0 [3]	Can be implied by graph or in writing. Ignore lines extended
(iii) $gf(x) = (2x+3)^2 - 6(2x+3)$ $4x^2 - 9$ $4x^2 - 9 < 16 \Rightarrow x^2 < \frac{25}{4}$ $-\frac{5}{2} \le x \le 0$	M1 A1 M1 A1A1 [5]	 Solving any quadratic to do with f and g <16, to x = Condone < and >

Q17.

10 **(i)** $4(x-3)^2 - 25$ Vertex is $(3, -25)$	B1B1B1 B1√ [4]	Or $a = 4$ $b = 3$, $c = -25$ ft to *their* (b, c). Accept if not 'hence'
(ii) range is (g(x)) > –9 Allow >	B1B1 [2]	B1 for > , B1 for –9 Accept e.g. $[-9,\infty]$
(iii) $(x-3)^2 = \frac{1}{4(y+25)}$ $x - 3 = \frac{(\pm)1}{2}\sqrt{y+25}$ $g^{-1}(x) = 3 - \frac{1}{2}\sqrt{x+25}$ Domain is $x > -9$	M1 DM1 A1 B1√ [4]	 Attempt to square root both sides cao ft from *their* **(ii)**

Q18.

6 **(i)** $\cos\left(\frac{1}{2}x + \frac{\pi}{6}\right)$ (= 1) $x = ! \frac{\pi}{3}$	B1 B1 [2]	 cao
(ii) $\frac{1}{2}\cos x + \frac{\pi}{6} = 1$ $\cos x = 0.9528$ $x = \pm 0.31$	B1 B1 B1 B1√ [4]	 √ for negative of first answer

Q19.

$11 - x^2 = 5 - x \Rightarrow x^2 - x - 6 (= 0)$	B1	oe
	B1 B1 [3]	cao
7 **(i)** $p = 3; \quad q = 2$	B1 B1 B1	
	B1 B1 [5]	
(ii)		

Q20.

5 **(i)** $x = (\pm)\sqrt{y-1}$	**B1**	**OR** $y^2 = x - 1$ (x/y interchange 1[st])
$f^{-1} : x \mapsto \sqrt{x-1}$ for $x > 1$	**B1B1** **[3]**	
(ii) $ff(x) = (x^2+1)^2 + 1$	**B1**	Or $x^4 + 2x^2 - (153/16) = 0$
$x^2 + 1 = (\pm)13/4$	**M1**	Or $x^2 = 9/4, (-17/4)$
$x = 3/2$	**A1** **[3]**	www. Condone $\pm 3/2$
Alt.(ii) $f(x) = f^{-1}(185/16) = 13/4$ **M1**		**Alt.**(ii) $f(3/2) = 13/4$ **B1**
$x = f^{-1}(13/4)$ **M1**		$f(13/4) = 185/16$ **B1**
$x = 3/2$ **A1**		$x = 3/2$ **B1**
		SC.B2 answer 1.5 with no working

Q21.

10 **(i)** Range is (y) $> c^2 + 4c$	B1	Allow $\geq$
$x^2 + 4x = (x+2)^2 - 4$	M1	**OR** $\frac{dy}{dx} = 2x + 4 = 0$
(Smallest value of c is) -2	A1 [3]	-2 with no (wrong) working gets B2
(ii) $5a + b = 11$	B1	
$(a+b)^2 + 4(a+b) = 21$	B1	
$(11 - 5a + a)^2 + 4(11 - 5a + a) = 21$	M1	**OR** corresponding equation in b
$(8)(2a^2 - 13a + 18) = (8)(2a - 9)(a - 2) = 0$	M1	**OR** $(8)(2b + 23)(b - 1) = 0$
$a = \frac{9}{2}, 2$ OR $b = \left(-\frac{23}{2}\right), 1$	A1	A1 for either a or b correct. Condone 2nd
	A1 [6]	value. Spotted solution scores only B marks.
Alt. **(ii)** Last 5 marks $f^{-1}(x) = \sqrt{x+4} - 2$ B1 $g(1) = f^{-1} = (21)$ used M1 $a + b = \sqrt{25} - 2 = 3$ A1 Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2, b = 1$ A1		**Alt. (ii)** Last 4 marks $(a + b + 7)(a + b - 3) = 0$ M1A1 (Ignore solution involving $a + b = -7$) Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2, b = 1$ A1