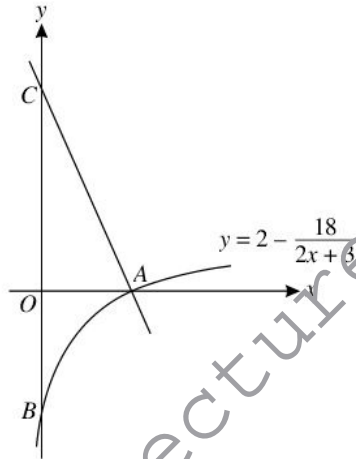


Q1.

- 6 A curve is such that $\frac{dy}{dx} = 3x^2 - 6$ and the point (9, 2) lies on the curve.
- (i) Find the equation of the curve. [4]
 - (ii) Find the x -coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

Q2.

7



The diagram shows part of the curve $y = 2 - \frac{18}{2x+3}$, which crosses the x -axis at A and the y -axis at B. The normal to the curve at A crosses the y -axis at C.

- (i) Show that the equation of the line AC is $9x + 4y = 27$. [6]
- (ii) Find the length of BC. [2]

Q3.

- 2 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

Q4.

- 6 The variables x , y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

- (i) Show that $z = 3x + \frac{1200}{x}$. [1]
- (ii) Find the stationary value of z and determine its nature. [6]

Q5.

- 4 (a) Differentiate $\frac{2x^3+5}{x}$ with respect to x . [3]
- (b) Find $\int (3x-2)^5 dx$ and hence find the value of $\int_0^1 (3x-2)^5 dx$. [4]

Q6.

- 4 A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius, r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of k and the rate at which the mass is increasing on this day. [5]

Q7.

- 10 It is given that a curve has equation $y = f(x)$, where $f(x) = x^3 - 2x^2 + x$.
- (i) Find the set of values of x for which the gradient of the curve is less than 5. [4]
- (ii) Find the values of $f(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

Q8.

- 7 The curve $y = \frac{10}{2x+1} - 2$ intersects the x -axis at A . The tangent to the curve at A intersects the y -axis at C .
- (i) Show that the equation of AC is $5y + 4x = 8$. [5]
- (ii) Find the distance AC . [2]

Q9.

- 1 It is given that $f(x) = (2x-5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

Q10.

- 9 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.
- (i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]
- (ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]
- (iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

Q11.

- 6 The non-zero variables x , y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

Q12.

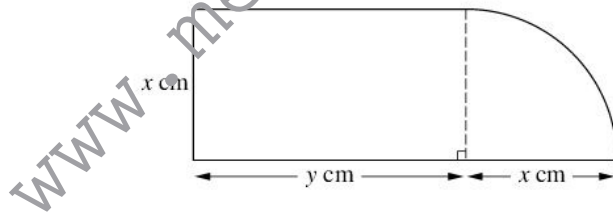
- 4 The equation of a curve is $y = x^4 + 4x + 9$.
- (i) Find the coordinates of the stationary point on the curve and determine its nature. [4]
- (ii) Find the area of the region enclosed by the curve, the x -axis and the lines $x = 0$ and $x = 1$. [3]

Q13.

- 7 The equation of a curve is $y = \frac{12}{x^2 + 3}$.
- (i) Obtain an expression for $\frac{dy}{dx}$. [2]
- (ii) Find the equation of the normal to the curve at the point $P(1, 3)$. [3]
- (iii) A point is moving along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.012 units per second. Find the rate of change of the y -coordinate as the point passes through P . [2]

Q14.

8



The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

- (i) Express y in terms of x . [2]
- (ii) Show that the area of the plate, A cm², is given by $A = 30x - x^2$. [2]
- Given that x can vary,
- (iii) find the value of x at which A is stationary. [2]
- (iv) find this stationary value of A , and determine whether it is a maximum or a minimum value. [2]

Q15.

- 10** The equation of a curve is $y = 3 + 4x - x^2$.
- (i) Show that the equation of the normal to the curve at the point $(3, 6)$ is $2y = x + 9$. [4]
 - (ii) Given that the normal meets the coordinate axes at points A and B , find the coordinates of the mid-point of AB . [2]
 - (iii) Find the coordinates of the point at which the normal meets the curve again. [4]

Q16.

- 5** A curve has equation $y = \frac{1}{x-3} + x$.
- (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]
 - (ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]

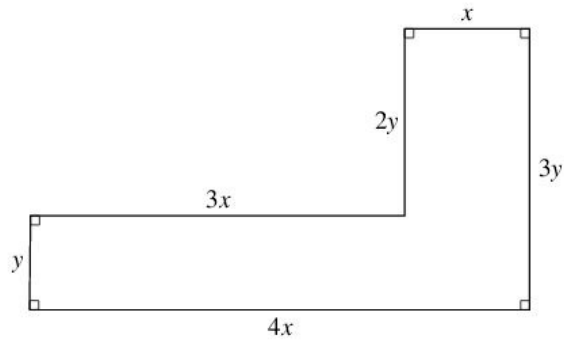
Q17.

- 4** A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geq -4$.
- (i) State the value of x for which $f(x)$ has a stationary value. [1]
 - (ii) Find an expression for $f(x)$ in terms of x . [4]

Q18.



7



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48m.

- (i) Find an expression for y in terms of x . [1]
- (ii) Given that the area of the garden is $A \text{ m}^2$, show that $A = 48x - 8x^2$. [2]
- (iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

Q19.

- 8 A curve $y = f(x)$ has a stationary point at $P(3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where k is a constant.
- (i) Show that $k = -2$ and hence find the x -coordinate of the other stationary point, Q . [4]
 - (ii) Find $f''(x)$ and determine the nature of each of the stationary points P and Q . [2]
 - (iii) Find $f(x)$. [4]

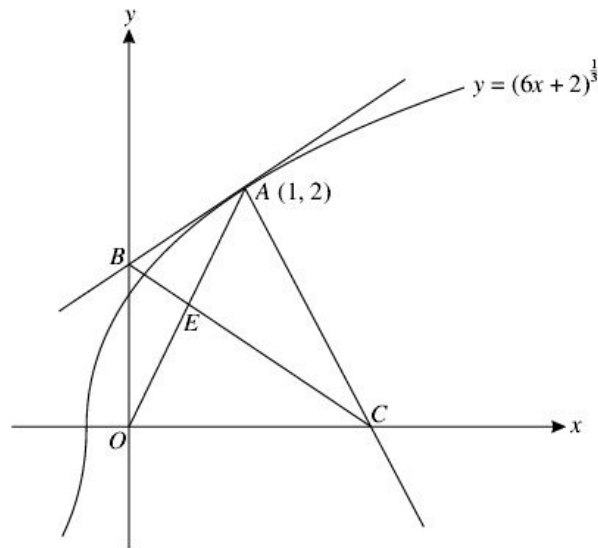
Q20.

Q21.

- 5 A curve has equation $y = 2x + \frac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. [5]

Q22.

11



The diagram shows the curve $y = (6x + 2)^{\frac{1}{3}}$ and the point $A(1, 2)$ which lies on the curve. The tangent to the curve at A cuts the y -axis at B and the normal to the curve at A cuts the x -axis at C .

- (i) Find the equation of the tangent AB and the equation of the normal AC . [5]
- (ii) Find the distance BC . [3]
- (iii) Find the coordinates of the point of intersection, E , of OA and BC , and determine whether E is the mid-point of OA . [4]

Q23.

- 2 It is given that $f(x) = \frac{1}{x^3} - x^3$, for $x > 0$. Show that f is a decreasing function. [3]

Q24.

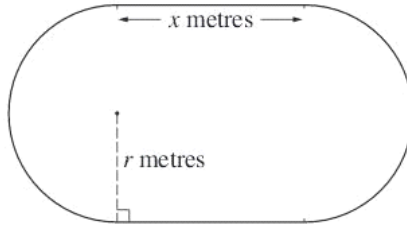
8 A curve is such that

$$\frac{dy}{dx} = 2(3x + 4)^{\frac{3}{2}} - 6x - 8.$$

- (i) Find $\frac{d^2y}{dx^2}$. [2]
- (ii) Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]
- (iii) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. [5]

Q25.

8



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

- (i) Show that the area, A m², of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]
- (ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

Q26.

- 9 A curve has equation $y = \frac{k}{x+2} + x$, where k is a positive constant. Find, in terms of k , the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

