Online Classes : Megalecture@gmail.com
www.youtube.com/megalecture
www.megalecture.com

These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

6 The equation of a curve is $y = \dfrac{1}{1 + \tan x}$.

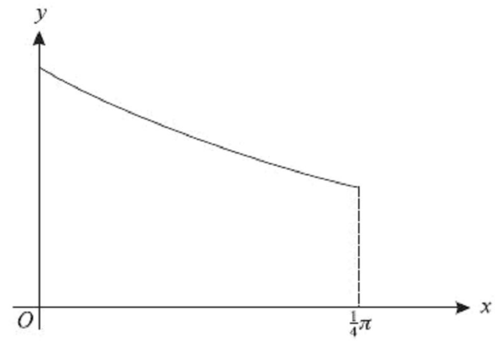
(i) Show, by differentiation, that the gradient of the curve is always negative. [4]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 + \tan x}\,dx,$$

giving your answer correct to 2 significant figures. [3]

(iii)

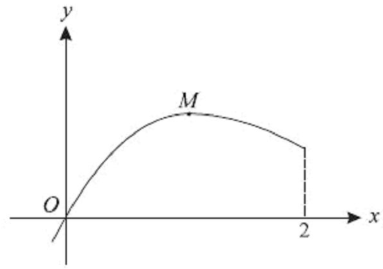


The diagram shows a sketch of the curve for $0 \leqslant x \leqslant \frac{1}{4}\pi$. State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). [1]

Q2.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

5



The diagram shows the part of the curve $y = xe^{-x}$ for $0 \leqslant x \leqslant 2$, and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_0^2 xe^{-x}\,dx,$$

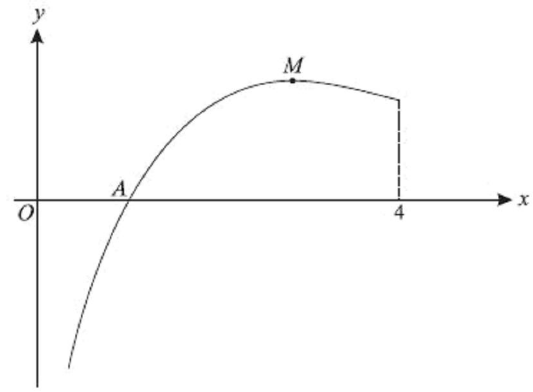
giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

Q3.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

6



The diagram shows the part of the curve $y = \frac{\ln x}{x}$ for $0 < x \leqslant 4$. The curve cuts the x-axis at A and its maximum point is M.

(i) Write down the coordinates of A. [1]

(ii) Show that the x-coordinate of M is e, and write down the y-coordinate of M in terms of e. [5]

(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_1^4 \frac{\ln x}{x}\,dx,$$

correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

Q4.

7 **(i)** Differentiate $\ln(2x + 3)$. [2]

(ii) Hence, or otherwise, show that

$$\int_{-1}^{3} \frac{1}{2x+3}\,dx = \ln 3.$$ [3]

(iii) Find the quotient and remainder when $4x^2 + 8x$ is divided by $2x + 3$. [3]

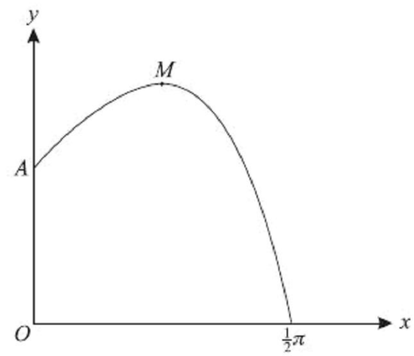
(iv) Hence show that

$$\int_{-1}^{3} \frac{4x^2 + 8x}{2x+3}\,dx = 12 - 3\ln 3.$$ [3]

Q5.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

7



The diagram shows the part of the curve $y = e^x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curve meets the y-axis at the point A. The point M is a maximum point.

(i) Write down the coordinates of A. [1]

(ii) Find the x-coordinate of M. [4]

(iii) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} e^x \cos x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(iv) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(iii)**. [1]

Q6.

8 (a) Find the equation of the tangent to the curve $y = \ln(3x - 2)$ at the point where $x = 1$. [4]

(b) **(i)** Find the value of the constant A such that

$$\frac{6x}{3x-2} \equiv 2 + \frac{A}{3x-2}.$$ [2]

(ii) Hence show that $\int_2^6 \frac{6x}{3x-2} \, dx = 8 + \frac{8}{3}\ln 2$. [5]

Q7.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

8 (i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) By expressing $\cot^2 x$ in terms of $\operatorname{cosec}^2 x$ and using the result of part (i), show that

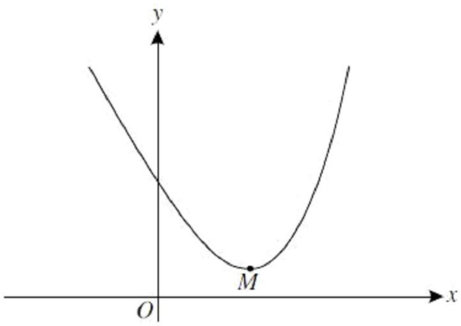
$$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx = 1 - \tfrac{1}{4}\pi.$$ [4]

(iii) Express $\cos 2x$ in terms of $\sin^2 x$ and hence show that $\frac{1}{1 - \cos 2x}$ can be expressed as $\frac{1}{2}\operatorname{cosec}^2 x$. Hence, using the result of part (i), find

$$\int \frac{1}{1 - \cos 2x} \, dx.$$ [3]

Q8.

5



The diagram shows the curve $y = 4e^{\frac{1}{2}x} - 6x + 3$ and its minimum point M.

(i) Show that the x-coordinate of M can be written in the form $\ln a$, where the value of a is to be stated. [5]

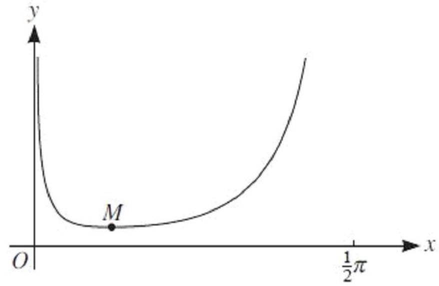
(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. [4]

Q9.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

7 (a) Find the exact area of the region bounded by the curve $y = 1 + e^{2x-1}$, the x-axis and the lines $x = \frac{1}{2}$ and $x = 2$. [4]

(b)



The diagram shows the curve $y = \dfrac{e^{2x}}{\sin 2x}$ for $0 < x < \frac{1}{2}\pi$, and its minimum point M. Find the exact x-coordinate of M. [5]

Q10.

7 (i) By differentiating $\dfrac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\dfrac{dy}{dx} = -\operatorname{cosec}^2 x$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \operatorname{cosec}^2 x \, dx = \sqrt{3}$. [2]

By using appropriate trigonometrical identities, find the exact value of

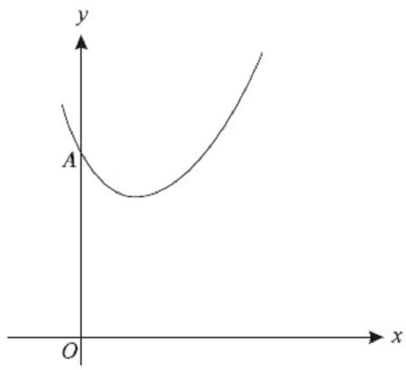
(iii) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cot^2 x \, dx$, [3]

(iv) $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{1 - \cos 2x} \, dx$. [3]

Q11.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

7



The diagram shows the curve $y = 2e^x + 3e^{-2x}$. The curve cuts the y-axis at A.

(i) Write down the coordinates of A. [1]

(ii) Find the equation of the tangent to the curve at A, and state the coordinates of the point where this tangent meets the x-axis. [6]

(iii) Calculate the area of the region bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = 1$, giving your answer correct to 2 significant figures. [4]

Q12.

6 A curve is such that $\frac{dy}{dx} = e^{2x} - 2e^{-x}$. The point (0, 1) lies on the curve.

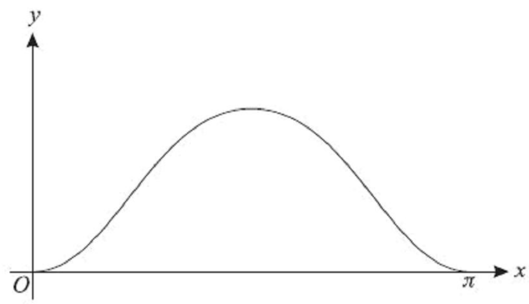
(i) Find the equation of the curve. [4]

(ii) The curve has one stationary point. Find the x-coordinate of this point and determine whether it is a maximum or a minimum point. [5]

Q13.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

7



The diagram shows the part of the curve $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$.

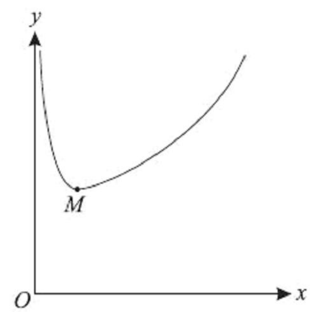
(i) Show that $\frac{dy}{dx} = \sin 2x$. [2]

(ii) Hence find the x-coordinates of the points on the curve at which the gradient of the curve is 0.5. [3]

(iii) By expressing $\sin^2 x$ in terms of $\cos 2x$, find the area of the region bounded by the curve and the x-axis between 0 and π. [5]

Q14.

6



The diagram shows the part of the curve $y = \frac{e^{2x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the coordinates of M. [5]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_1^2 \frac{e^{2x}}{x}\,dx,$$

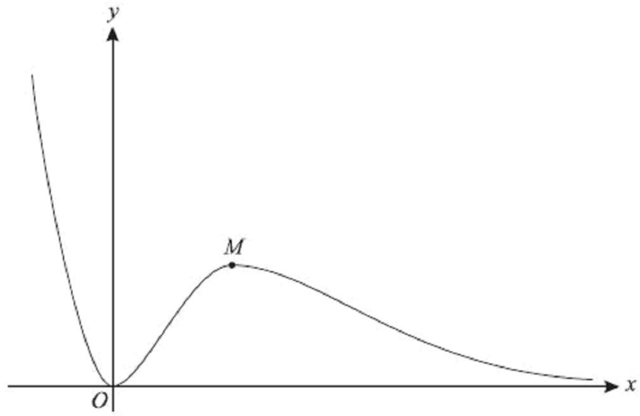
giving your answer correct to 1 decimal place. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

Q15.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

8



The diagram shows the curve $y = x^2e^{-x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Show that the tangent to the curve at the point where $x = 1$ passes through the origin. [3]

(iii) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_1^3 x^2e^{-x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

Q16.

8 **(i) (a)** Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1 + \sin x}{\cos^2 x}.$$

(b) Hence prove that

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1 - \sin x}.$$

[3]

(ii) By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [3]

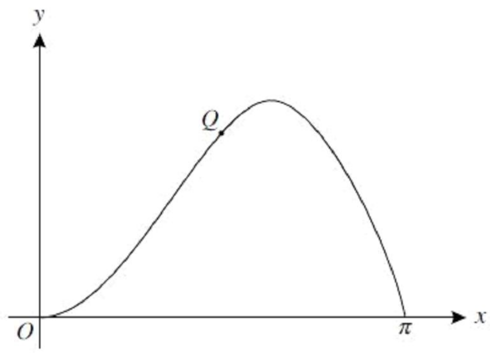
(iii) Using the results of parts **(i)** and **(ii)**, find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 - \sin x}\,dx.$$ [3]

Q17.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

8



The diagram shows the curve $y = x\sin x$, for $0 \leqslant x \leqslant \pi$. The point $Q\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ lies on the curve.

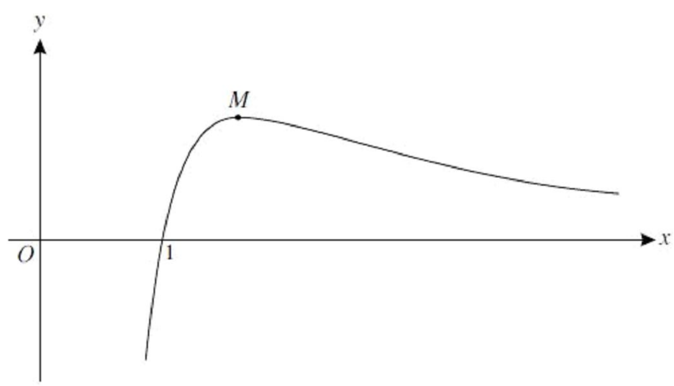
(i) Show that the normal to the curve at Q passes through the point $(\pi, 0)$. [5]

(ii) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x - x\cos x)$. [2]

(iii) Hence evaluate $\displaystyle\int_0^{\frac{1}{2}\pi} x\sin x\,\mathrm{d}x$. [3]

Q18.

7



The diagram shows the curve $y = \dfrac{\ln x}{x^2}$ and its maximum point M.

(i) Find the exact coordinates of M. [5]

(ii) Use the trapezium rule with three intervals to estimate the value of

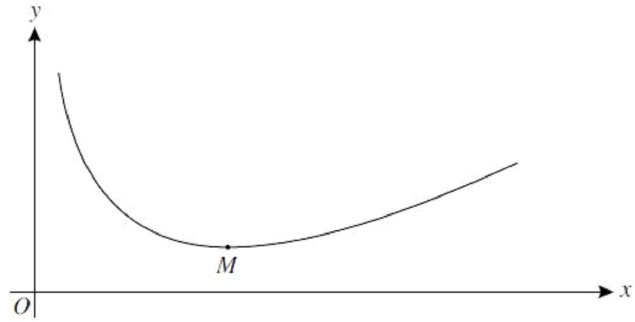
$$\int_1^4 \frac{\ln x}{x^2}\,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

Q19.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

3



The diagram shows the curve $y = x - 2\ln x$ and its minimum point M.

(i) Find the x-coordinate of M. [2]

(ii) Use the trapezium rule with three intervals to estimate the value of

$$\int_2^5 (x - 2\ln x)\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). [1]

Q20.

8 (i) By differentiating $\frac{1}{\cos\theta}$, show that if $y = \sec\theta$ then $\frac{dy}{d\theta} = \tan\theta\sec\theta$. [3]

(ii) Hence show that

$$\frac{d^2y}{d\theta^2} = a\sec^3\theta + b\sec\theta,$$

giving the values of a and b. [4]

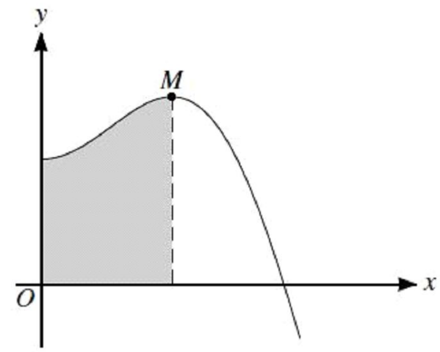
(iii) Find the exact value of

$$\int_0^{\frac{1}{4}\pi} (1 + \tan^2\theta - 3\sec\theta\tan\theta)\,d\theta.$$ [5]

Q21.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

5



The diagram shows part of the curve

$$y = 2\cos x - \cos 2x$$

and its maximum point M. The shaded region is bounded by the curve, the axes and the line through M parallel to the y-axis.

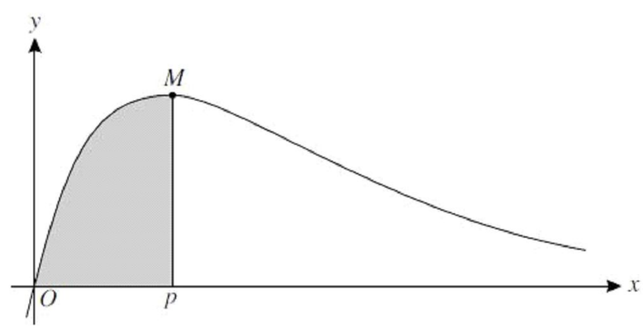
(i) Find the exact value of the x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region. [4]

P3 (variant1 and 3)

Q1.

5



The diagram shows the curve $y = e^{-x} - e^{-2x}$ and its maximum point M. The x-coordinate of M is denoted by p.

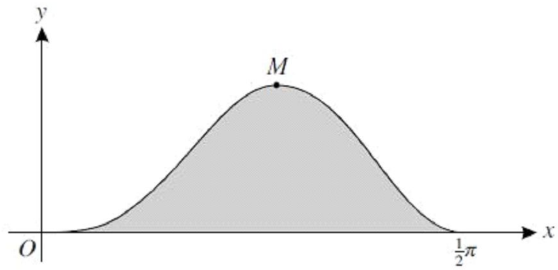
(i) Find the exact value of p. [4]

(ii) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = p$ is equal to $\frac{1}{8}$. [4]

Q2.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

8



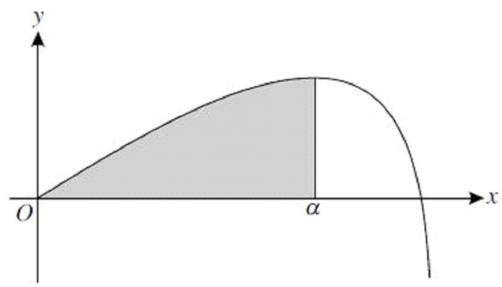
The diagram shows the curve $y = 5\sin^3 x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [5]

(ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [5]

Q3.

5



The diagram shows the curve

$$y = 8\sin\tfrac{1}{2}x - \tan\tfrac{1}{2}x$$

for $0 \leqslant x < \pi$. The x-coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

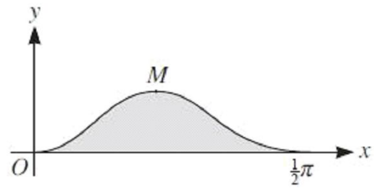
(i) Show that $\alpha = \frac{2}{3}\pi$. [3]

(ii) Find the exact value of the area of the shaded region. [4]

Q4.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

9



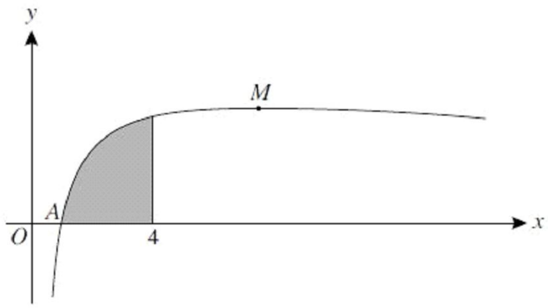
The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [4]

Q5.

9



The diagram shows the curve $y = \dfrac{\ln x}{\sqrt{x}}$ and its maximum point M. The curve cuts the x-axis at the point A.

(i) State the coordinates of A. [1]

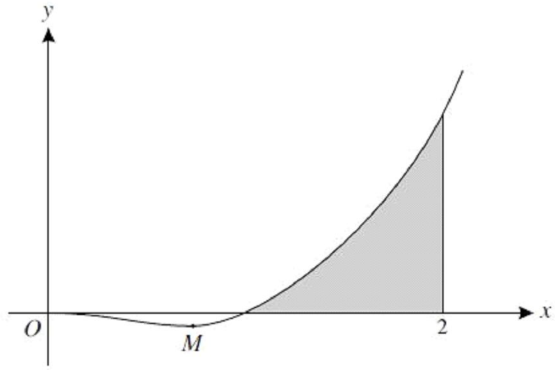
(ii) Find the exact value of the x-coordinate of M. [4]

(iii) Using integration by parts, show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 4$ is equal to $8 \ln 2 - 4$. [5]

Q6.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

9



The diagram shows the curve $y = x^3 \ln x$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [5]

Q7.

4 It is given that $\mathrm{f}(x) = 4\cos^2 3x$.

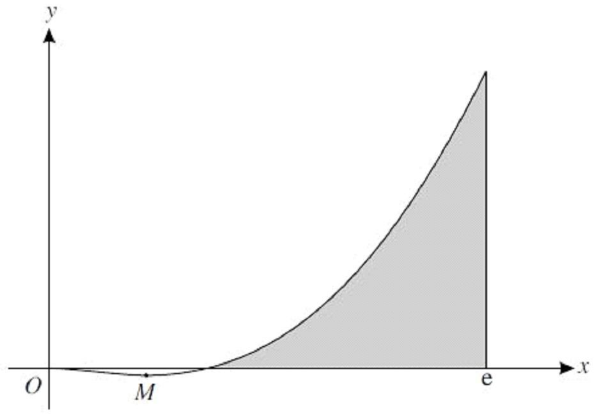
(i) Find the exact value of $\mathrm{f}'(\frac{1}{9}\pi)$. [3]

(ii) Find $\int \mathrm{f}(x)\,\mathrm{d}x$. [3]

Q8.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

9



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

Q9.

5 (i) By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec x \tan x$. [2]

(ii) Show that $\frac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\frac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

(iv) Hence show that $\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\,\mathrm{d}x = \frac{1}{4}(8\sqrt{2} - \pi)$. [3]

Q10.

5 The expression $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3x\mathrm{e}^{-2x}$.

(i) Find the exact value of $\mathrm{f}'\left(-\frac{1}{2}\right)$. [3]

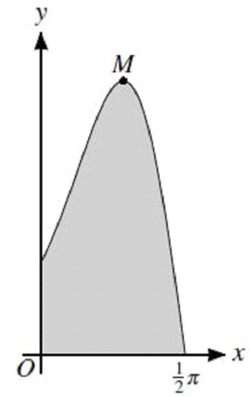
(ii) Find the exact value of $\int_{-\frac{1}{2}}^{0} \mathrm{f}(x)\,\mathrm{d}x$. [5]

Q11.

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

Online Classes : Megalecture@gmail.com
www.youtube.com/megalecture
www.megalecture.com

9



The diagram shows the curve $y = e^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com

www.youtube.com/megalecture
MEGA LECTURE
www.megalecture.com