Q1.

6 (i)	Attempt to apply the chain or quotient rule	M1	
	Obtain derivative of the form $\dfrac{k\sec^2 x}{(1+\tan x)^2}$ or equivalent	A1	
	Obtain correct derivative $-\dfrac{\sec^2 x}{(1+\tan x)^2}$ or equivalent	A1	
	Explain why derivative, and hence gradient of the curve, is always negative	A1	
		[4]	
(ii)	State or imply correct ordinates: 1, 0.7071.., 0.5	B1	
	Use correct formula, or equivalent, with $h = \frac{1}{8}\pi$ and three ordinates	M1	
	Obtain answer 0.57 (0.57220...) ± 0.01 (accept 0.18π)	A1	
		[3]	

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(iii)	Justify the statement that the rule gives an over-estimate	B1	
		[1]	

Q2.

5	(i)	State derivative of the form $(e^{-x} \pm xe^{-x})$. Allow $xe^x \pm e^x$ {via quotient rule}	M1	
		Obtain correct derivative of $e^{\pm x} - xe^{-x}$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer x = 1	A1	4
	(ii)	Show or imply correct ordinates 0, 0.367879…, 0.27067…	B1	
		Use correct formula, or equivalent, with h = 1 and three ordinates	M1	
		Obtain answer 0.50 with no errors seen	A1	3
	(iii)	Justify statement that the rule gives an under-estimate	B1	1

Q3.

6	(i)	State coordinates (1, 0)	B1	1
	(ii)	Use quotient or product rule	M1	
		Obtain correct derivative, e.g. $\frac{-\ln x}{x^2} + \frac{1}{x^2}$	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = e$	A1	
		Obtain $y = \frac{1}{e}$	A1	5
	(iii)	Show or imply correct coordinates 0, 0.34657..., 0.36620..., 0.34657,,,	B1	
		Use correct formula, or equivalent, with $h = 1$ and four ordinates	A1	
		Obtain answer 0.89 with no errors seen	A1	3
	(iv)	Justify statement that the rule gives an under-estimate	B1	1

Q4.

7	(i)	Obtain derivative of the form $\frac{k}{2x+3}$, where $k = 2$ or $k = 1$	M1	
		Obtain correct derivative $\frac{2}{2x+3}$	A1	2
	(ii)	State indefinite integral of the form $m \ln(2x+3)$	M1*	
		Use limits correctly	M1(dep*)	
		Obtain given answer	A1	3
	(iii)	Carry out division method reaching a linear quotient and constant remainder	M1	
		Obtain quotient $2x + 1$	A1	
		Obtain remainder -3	A1	3
	(iv)	Attempt integration of an integrand of the form $ax + b + \frac{c}{2x+3}$	M1	
		Obtain indefinite integral $x^2 + x - \frac{3}{2}\ln(2x+3)$	A1√	
		Substitute limits and obtain given answer	A1	3
		[The f.t. mark is also available if the indefinite integral of the third term is omitted but its definite integral is stated to be $c \ln 3$.]		

Q5.

7	(i)	State coordinates (0, 1) for A	B1	[1]
	(ii)	Differentiate using the product rule	M1*	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1*	
		Obtain $x = \frac{1}{4}\pi$ or 0.785 (allow 45°)	A1	[4]
	(ii)	Show or imply correct ordinates 1, 1.4619…, 1.4248…, 0	B1	
		Use correct formula or equivalent with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain correct answer 1.77 with no errors seen	A1	[3]
	(iv)	Justify statement that the trapezium rule gives and underestimate	B1	[1]

Q6.

8 (a)	State derivative is $k/(3x-2)$ where $k = 3.1$, or $\frac{1}{3}$	M1	
	State correct derivative $3/(3x-2)$	A1	
	Form the equation of the tangent at the point where $x = 1$	M1	
	Obtain answer $y = 3x - 3$, or equivalent	A1	[4]
(b) (i)	Carry out a complete method for finding A	M1	
	Obtain $A = 4$	A1	[2]
(ii)	Integrate and obtain term $2x$	B1	
	Obtain second term of the form $a\ln(3x-2)$	M1	
	Obtain second term $\frac{4}{3}\ln(3x-2)$	A1√	
	Substitute limits correctly	M1	
	Obtain given answer following full and correct working	A1	[5]

Q7.

8 (i)	Use quotient rule	M1	
	Obtain correct derivative in any form	A1	
	Obtain given result correctly	A1	[3]
(ii)	State $\cot^2 x \equiv -1 + \operatorname{cosec}^2 x$, or equivalent	B1	
	Obtain integral $-x - \cot x$ (f.t. on signs in the identity)	B1√	
	Substitute correct limits correctly	M1	
	Obtain given answer	A1	[4]
(iii)	Use trig formulae to convert integrand to $\dfrac{1}{k\sin^2 x}$ where $k = \pm 2$, or ± 1	M1	
	Obtain given answer $\frac{1}{2}\operatorname{cosec}^2 x$ correctly	A1	
	Obtain answer $-\frac{1}{2}\cot x + c$, or equivalent	B1	[3]

Q8.

5 (i)	Differentiate to obtain expression of form $ke^{\frac{1}{2}x} + m$	M1	
	Obtain correct $2e^{\frac{1}{2}x} - 6$	A1	
	Equate attempt at first derivative to zero and attempt solution	DM1	
	Obtain $\frac{1}{2}x = \ln 3$ or equivalent	A1	
	Conclude $x = \ln 9$ or $a = 9$	A1	[5]
(ii)	Integrate to obtain expression of form $ae^{\frac{1}{2}x} + bx^2 + cx$	M1	
	Obtain correct $8e^{\frac{1}{2}x} - 3x^2 + 3x$	A1	
	Substitute correct limits and attempt simplification	DM1	
	Obtain $8e - 14$	A1	[4]

Q9.

7	**(a)**	Obtain one term of form ke^{2x-1} with any non-zero k	M1	
		Obtain correct integral $x + \frac{1}{2}e^{2x-1}$	A1	
		Substitute limits, giving exact values	M1	
		Correct answer $\frac{1}{2}e^3 + 1$	A1	[4]
	(b)	Use product or quotient rule	M1*	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1* dep	
		Obtain $\tan 2x = 1$	A1	
		Obtain $x = \frac{\pi}{8}$	A1	[5]

Q10.

7 (i)	Attempt to differentiate using the quotient, product or chain rule	M1
	Obtain derivative in any correct form	A1
	Obtain the given answer correctly	A1
		[3]

(ii)	State or imply the indefinite integral is $-\cot x$	B1
	Substitute limits and obtain given answer correctly	B1
		[2]
(iii)	Use $\cot^2 x = \operatorname{cosec}^2 x - 1$ and attempt to integrate both terms, or equivalent	M1
	Substitute limits where necessary and obtain a correct unsimplified answer	A1
	Obtain final answer $\sqrt{3} - \frac{1}{3}\pi$	A1
		[3]
(iv)	Use $\cos 2A$ formula and reduce denominator to $2\sin^2 x$	B1
	Use given result and obtain answer of the form $k\sqrt{3}$	M1
	Obtain correct answer $\frac{1}{2}\sqrt{3}$	A1
		[3]

Q11.

7 (i)	State coordinates (0, 5)	B1	1
(ii)	State first derivative of the form $k\,e^x + m\,e^{-2x}$, where $km \neq 0$	M1	
	Obtain correct first derivative $2\,e^x - 6\,e^{-2x}$	A1	
	Substitute $x = 0$, obtaining gradient of -4	A1√	
	Form equation of line through A with this gradient (NOT the normal)	M1	
	Obtain equation in any correct form e.g. $y - 5 = -4x$	A1	
	Obtain coordinates (1.25, 0) or equivalent	A1	6
(iii)	Integrate and obtain $2\,e^x - \frac{3}{2}e^{-2x}$, or equivalent	B1 + B1	
	Use limits $x = 0$ and $x = 1$ correctly	M1	
	Obtain answer 4.7	A1	4

Q12.

6 (i)	State $\frac{1}{2}e^{2x}$ as integral of e^{2x}	B1	
	State $y = \frac{1}{2}e^{2x} + 2e^{-x} + c$	B1	
	Evaluate c	M1	
	Obtain answer $y = \frac{1}{2}e^{2x} + 2e^{-x} - 1\frac{1}{2}$	A1	4
	[Condone omission of c for the second B1.]		
(ii)	Equate derivative to zero	M1	
	EITHER: Obtain $e^{3x} = 2$	A1	
	Use logarithms and obtain a linear equation in x	M1	
	Obtain answer $x = 0.231$	A1	
	Show that the point is a minimum with no errors seen	A1	
	OR: Use logarithms and obtain a linear equation in x	M1	
	Obtain $2x = \ln 2 - x$	A1	
	Obtain answer $x = 0.231$	A1	
	Show that the point is a minimum with no errors seen	A1 √	5

Q13.

7 (i)	Differentiate using the chain or product rule	M1	
	Obtain given answer correctly	A1	2
(ii)	Use correct method for solving $\sin 2x = 0.5$	M1	
	Obtain answer $x = \frac{1}{12}\pi$ (or 0.262 radians)	A1	
	Obtain answer $x = \frac{5}{12}\pi$ (or 1.31 radians) and no others in range	A1	3
(iii)	Replace integrand by $\frac{1}{2} - \frac{1}{2}\cos 2x$, or equivalent	B1	
	Integrate and obtain $\frac{1}{2}x - \frac{1}{4}\sin 2x$, or equivalent	B1√ + B1√	
	Use limits $x = 0$ and $x = \pi$ correctly	M1	
	Obtain final answer 1.57 (or $\frac{1}{2}\pi$)	A1	5

Q14.

6 (i)	Use quotient or product rule		M1	
	Obtain derivative in any correct form, e.g. $e^{2x}\left(\frac{2}{x}-\frac{1}{x^2}\right)$		A1	
	Equate derivative to zero and solve for x		M1	
	Obtain $x=\frac{1}{2}$		A1	
	Obtain $y = 2e$ (or 5.44)	(* allow $\sqrt{y}=2e$ if $x=\frac{1}{2}$	A1*	5
(ii)	Show or imply correct ordinates 7.389..., 13.390..., 27.299...		B1	
	Use correct formula, or equivalent, with $h = 0.5$ and three ordinates	'illicitly' obtained)	M1	
	Obtain answer 15.4 with no errors seen		A1	3
(iii)	Justify the statement that the rule gives an over-estimate		B1	1

Q15.

8 (i)	Differentiate using product or quotient rule	M1	
	Obtain derivative in any correct form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ correctly, with no other solution	A1	[4]
(ii)	Find the gradient of the curve when $x = 1$, must be simplified, allow 0.368	B1	
	Form the equation of the tangent when $x = 1$	M1	
	Show that it passes through the origin	A1	[3]
(iii)	State or imply correct ordinates 0.36787…, 0.54134…, 0.44808…	B1	
	Use correct formula, or equivalent, correctly with $h = 1$ and three ordinates	M1	
	Obtain answer 0.95 with no errors seen	A1	[3]

Q16.

8 (i) (a)	Use trig formulae and justify given result	B1	
(b)	Use $1-\sin^2 x = \cos^2 x$	M1	
	Obtain given result correctly	A1	[3]
(ii)	Use quotient or chain rule	M1	
	Obtain correct derivative in any form	A1	
	Obtain given result correctly	A1	[3]
(iii)	Obtain integral $\tan x + \sec x$	B1	
	Substitute limits correctly	M1	
	Obtain exact answer $\sqrt{2}$, or equivalent	A1	[3]

Q17.

8	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Substitute $x = \frac{1}{2}\pi$, and obtain gradient of -1 for normal	A1√	
		from $y' = \sin x - x\cos x$ ONLY		
		Show that line through $\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ with gradient -1 passes through $(\pi, 0)$	M1	
			A1	[5]
	(ii)	Differentiate $\sin x$ and use product rule to differentiate $x\cos x$	M1	
		Obtain $x\sin x$, or equivalent	A1	[2]
	(iii)	State that integral is $\sin x - x\cos x(+c)$	B1	
		Substitute limits 0 and $\frac{\pi}{2}$ correctly	M1	
		Obtain answer 1	A1	[3]
		S. R. Feeding limits into original <u>integrand</u>, 0/3		

Q18.

7	**(i)**	Use product or quotient rule	M1*	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1*(dep)	
		Obtain $x = e^{0.5}$ or $\sqrt{e}$	A1	
		Obtain $\frac{1}{2e}$, or equivalent	A1	[5]
	(ii)	State or imply correct ordinates 0, 0.17328..., 0.12206..., 0.08664...	B1	
		Use correct formula, or equivalent, correctly with $h = 1$ and four ordinates	M1	
		Obtain answer 0.34 with no errors seen	A1	[3]

Q19.

3	**(i)**	Obtain correct derivative	B1	
		Obtain $x = 2$ only	B1	[2]
	(ii)	State or imply correct ordinates 0.61370..., 0.80277..., 1.22741..., 1.78112...	B1	
		Use correct formula, or equivalent, correctly with $h = 1$ and four ordinates	M1	
		Obtain answer 3.23 with no errors seen	A1	[3]
	(iii)	Justify statement that the trapezium rule gives an over-estimate	B1	[1]

Q20.

8	**(i)**	Differentiate using chain or quotient rule	M1	
		Obtain derivative in any correct form	A1	
		Obtain given answer correctly	A1	[3]
	(ii)	Differentiate using product rule	M1	
		State derivative of $\tan\theta = \sec^2\theta$	B1	
		Use trig identity $1 + \tan^2\theta = \sec^2\theta$ correctly	M1	
		Obtain $2\sec^3\theta - \sec\theta$	A1	[4]
	(iii)	Use $\tan^2 x = \sec^2\theta - 1$ to integrate $\tan^2 x$	M1	
		Obtain $3\sec\theta$ from integration of $3\sec\theta\tan\theta$	B1	
		Obtain $\tan\theta - 3\sec\theta$	A1	
		Attempt to substitute limits, using exact values	M1	
		Obtain answer $4 - 3\sqrt{2}$	A1	[5]

Q21.

5	**(i)**	Differentiate to obtain $-2\sin x + 2\sin 2x$ or equivalent	B1	
		Use $\sin 2x = 2\sin x\cos x$ or equivalent	B1	
		Equate first derivative to zero and solve for x	M1	
		Obtain $\frac{1}{3}\pi$	A1	[4]

(ii)	Integrate to obtain form $k_1\sin x + k_2\sin 2x$	M1	
	Obtain correct $2\sin x - \frac{1}{2}\sin 2x$	A1	
	Apply limits 0 and their answer from part **(i)**	M1	
	Obtain $\frac{3}{4}\sqrt{3}$ or exact equivalent	A1	[4]

P3 (variant1 and 3)

Q1.

5	(i)	State derivative $-e^{-x} - (-2)e^{-2x}$, or equivalent	B1 + B1	
		Equate derivative to zero and solve for x	M1	
		Obtain $p = \ln 2$, or exact equivalent	A1	[4]
	(ii)	State indefinite integral $-e^{-x} - (-\frac{1}{2})e^{-2x}$, or equivalent	B1 + B1	
		Substitute limits $x = 0$ and $x = p$ correctly	M1	
		Obtain given answer following full and correct working	A1	[4]

Q2.

8	(i)	Use product and chain rule	M1	
		Obtain correct derivative in any form, e.g. $15\sin^2 x\cos^3 x - 10\sin^4 x\cos x$	A1	
		Equate derivative to zero and obtain a relevant equation in one trigonometric function	M1	
		Obtain $2\tan^2 x = 3$, $5\cos^2 x = 2$, or $5\sin^2 x = 3$	A1	
		Obtain answer $x = 0.886$ radians	A1	[5]
	(ii)	State or imply $\mathrm{d}u = -\sin x\,\mathrm{d}x$, or $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$, or equivalent	B1	
		Express integral in terms of u and $\mathrm{d}u$	M1	
		Obtain $\pm \int 5(u^2 - u^4)\,\mathrm{d}u$, or equivalent	A1	
		Integrate and use limits $u = 1$ and $u = 0$ (or $x = 0$ and $x = \frac{1}{2}\pi$)	M1	
		Obtain answer $\frac{2}{3}$, or equivalent, with no errors seen	A1	[5]

Q3.

5	(i)	Differentiate to obtain $4\cos\frac{1}{2}x - \frac{1}{2}\sec^2\frac{1}{2}x$	B1	
		Equate to zero and find value of $\cos\frac{1}{2}x$	M1	
		Obtain $\cos\frac{1}{2}x = \frac{1}{2}$ and confirm $\alpha = \frac{2}{3}\pi$	A1	[3]
	(ii)	Integrate to obtain $-16\cos\frac{1}{2}x \ldots$	B1	
		$\ldots + 2\ln\cos\frac{1}{2}x$ or equivalent	B1	
		Using limits 0 and $\frac{2}{3}\pi$ in $a\cos\frac{1}{2}x + b\ln\cos\frac{1}{2}x$	M1	
		Obtain $8 + 2\ln\frac{1}{2}$ or exact equivalent	A1	[4]

Q4.

9	(i)	Use product rule	M1	
		Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$	A1	
		Equate derivative to zero and use a double angle formula	M1*	
		Reduce equation to one in a single trig function	M1(dep*)	
		Obtain a correct equation in any form, e.g. $10\cos^3 x = 6\cos x$, $4 = 6\tan^2 x$ or $4 = 10\sin^2 x$	A1	
		Solve and obtain $x = 0.685$	A1	[6]
	(ii)	Using $du = \pm\cos x\, dx$, or equivalent, express integral in terms of u and du	M1	
		Obtain $\int 4u^2(1-u^2)\,du$, or equivalent	A1	
		Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	M1	
		Obtain answer $\frac{8}{15}$ (or 0.533)	A1	[4]

Q5.

9	(i)	State coordinates (1, 0)	B1	[1]
	(ii)	Use correct quotient or product rule	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = e^2$ correctly	A1	[4]

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(iii)	Attempt integration by parts reaching $a\sqrt{x}\ln x \pm a\int \sqrt{x}\frac{1}{x}dx$	M1*	
	Obtain $2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}dx$	A1	
	Integrate and obtain $2\sqrt{x}\ln x - 4\sqrt{x}$	A1	
	Use limits $x = 1$ and $x = 4$ correctly, having integrated twice	M1(dep*)	
	Justify the given answer	A1	[5]

Q6.

9	(i)	Use correct product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and find non-zero x	M1	
		Obtain $x = \exp(-\frac{1}{3})$, or equivalent	A1	
		Obtain $y = -1/(3e)$, or any ln-free equivalent	A1	[5]
	(ii)	Integrate and reach $kx^4 \ln x + l\int x^4 . \frac{1}{x} dx$	M1	
		Obtain $\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx$	A1	
		Obtain integral $\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$, or equivalent	A1	
		Use limits $x = 1$ and $x = 2$ correctly, having integrated twice	M1	
		Obtain answer $4\ln 2 - \frac{15}{16}$, or exact equivalent	A1	[5]

Q7.

4	(i)	Obtain derivative of form $k\cos 3x \sin 3x$, any constant k	M1	
		Obtain $-24\cos 3x \sin 3x$ or unsimplified equivalent	A1	
		Obtain $-6\sqrt{3}$ or exact equivalent	A1	[3]
	(ii)	Express integrand in the form $a + b\cos 6x$, where $ab \neq 0$	M1	
		Obtain $2 + 2\cos 6x$ o.e.	A1	
		Obtain $2x + \frac{1}{3}\sin 6x$ or equivalent, condoning absence of $+ c$, ft on a, b	A1√	[3]

Q8.

9	(i)	Use product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = e^{-\frac{1}{2}}$, or equivalent	A1	
		Obtain answer $y = -\frac{1}{2}e^{-1}$, or equivalent	A1	[5]
	(ii)	Attempt integration by parts reaching $kx^3 \ln x \pm k \int x^3 . \frac{1}{x} dx$	M1*	
		Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$, or equivalent	A1	
		Integrate again and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$, or equivalent	A1	
		Use limits $x = 1$ and $x = e$, having integrated twice	M1(dep*)	
		Obtain answer $\frac{1}{9}(2e^3 + 1)$, or exact equivalent	A1	[5]
		[SR: An attempt reaching $ax^2(x\ln x - x) + b\int 2x(x\ln x - x) dx$ scores M1. Then give the first A1 for $I = x^2(x\ln x - x) - 2I + \int 2x^2 dx$, or equivalent.]		

Q9.

5	**(i)**	Use correct quotient or chain rule	M1	
		Obtain the given answer correctly having shown sufficient working	A1	[2]
	(ii)	Use a valid method, e.g. multiply numerator and denominator by $\sec x + \tan x$, and a version of Pythagoras to justify the given identity	B1	[1]
	(iii)	Substitute, expand $(\sec x + \tan x)^2$ and use Pythagoras once	M1	
		Obtain given identity	A1	[2]
	(iv)	Obtain integral $2\tan x - x + 2\sec x$	B1	
		Use correct limits correctly in an expression of the form $a\tan x + bx + c\sec x$, or equivalent, where $abc \neq 0$	M1	
		Obtain the given answer correctly	A1	[3]

Q10.

5	**(i)**	Either	Use correct product rule	M1	
			Obtain $3e^{-2x} - 6xe^{-2x}$ or equivalent	A1	
			Substitute $-\frac{1}{2}$ and obtain 6e	A1	
		Or /	Take ln of both sides and use implicit differentiation correctly	M1	
			Obtain $\frac{dy}{dx} = y\left(\frac{1}{x} - 2\right)$ or equivalent	A1	
			Substitute $-\frac{1}{2}$ and obtain 6e	A1	[3]
	(ii)	Use integration by parts to reach $kxe^{-2x} \pm \int ke^{-2x}\,dx$		M1	
		Obtain $-\frac{3}{2}xe^{-2x} + \int \frac{3}{2}e^{-2x}\,dx$ or equivalent		A1	
		Obtain $-\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x}$ or equivalent		A1	
		Substitute correct limits correctly		DM1	
		Obtain $-\frac{3}{4}$ with no errors or inexact work seen		A1	[5]

Q11.

9	**(i)** Substitute for x and dx throughout using $u = \sin x$ and d$u = \cos x$ dx, or equivalent		M1	
	Obtain integrand e^{2u}		A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$		A1	
	Use limits $u = 0$, $u = 1$ correctly, or equivalent		M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent		A1	**5**

(ii) Use chain rule or product rule	M1	
Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
Solve a 3-term quadratic and obtain a value of x	M1	
Obtain answer 0.896	A1	**6**