P3 (variant1 and 3)

Q1.

7 The complex number $2 + 2i$ is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Sketch an Argand diagram showing the points representing the complex numbers 1, i and u. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1| \leqslant |z - i|$ and $|z - u| \leqslant 1$. [4]

(iii) Using your diagram, calculate the value of $|z|$ for the point in this region for which $\arg z$ is least. [3]

Q2.

8 (a) The equation $2x^3 - x^2 + 2x + 12 = 0$ has one real root and two complex roots. Showing your working, verify that $1 + i\sqrt{3}$ is one of the complex roots. State the other complex root. [4]

(b) On a sketch of an Argand diagram, show the point representing the complex number $1 + i\sqrt{3}$. On the same diagram, shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1 - i\sqrt{3}| \leqslant 1$ and $\arg z \leqslant \frac{1}{3}\pi$. [5]

Q3.

8 The complex number u is defined by $u = \dfrac{6 - 3i}{1 + 2i}$.

(i) Showing all your working, find the modulus of u and show that the argument of u is $-\frac{1}{2}\pi$. [4]

(ii) For complex numbers z satisfying $\arg(z - u) = \frac{1}{4}\pi$, find the least possible value of $|z|$. [3]

(iii) For complex numbers z satisfying $|z - (1 + i)u| = 1$, find the greatest possible value of $|z|$. [3]

Q4.

7 (i) Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

giving your answers in the form $x + iy$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

Q5.

4 The complex number u is defined by $u = \frac{(1+2\text{i})^2}{2+\text{i}}$.

(i) Without using a calculator and showing your working, express u in the form $x + \text{i}y$, where x and y are real. [4]

(ii) Sketch an Argand diagram showing the locus of the complex number z such that $|z - u| = |u|$. [3]

Q6.

10 **(a)** The complex numbers u and w satisfy the equations

$$u - w = 4\text{i} \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w, giving all answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2\text{i}| \leqslant 2$, $\arg z \leqslant -\frac{1}{4}\pi$ and $\text{Re}\, z \geqslant 1$, where $\text{Re}\, z$ denotes the real part of z. [5]

(ii) Calculate the greatest possible value of $\text{Re}\, z$ for points lying in the shaded region. [1]

Q7.

7 **(a)** Without using a calculator, solve the equation

$$3w + 2\text{i}w^* = 17 + 8\text{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\text{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\text{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\text{i}|$$

intersect at the point P. Express the complex number represented by P in the form $r\text{e}^{\text{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

Q8.

7 The complex number z is defined by $z = a + ib$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - ki)^* = z^* + ki$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10i| = 2|z - 4i|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2iz^* + 2iz - 12 = 0.$$

Hence show that $|z - 2i| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

Q9.

7 The complex number $-2 + i$ is denoted by u.

(i) Given that u is a root of the equation $x^3 - 11x - k = 0$, where k is real, find the value of k. [3]

(ii) Write down the other complex root of this equation. [1]

(iii) Find the modulus and argument of u. [2]

(iv) Sketch an Argand diagram showing the point representing u. Shade the region whose points represent the complex numbers z satisfying both the inequalities

$$|z| < |z - 2| \quad \text{and} \quad 0 < \arg(z - u) < \tfrac{1}{4}\pi.$$ [4]

Q10.

6 The complex number z is given by

$$z = (\sqrt{3}) + i.$$

(i) Find the modulus and argument of z. [2]

(ii) The complex conjugate of z is denoted by z^*. Showing your working, express in the form $x + iy$, where x and y are real,

(a) $2z + z^*$,

(b) $\dfrac{iz^*}{z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z and iz^* respectively. Prove that angle $AOB = \tfrac{1}{6}\pi$. [3]

Q11.

3 The complex number w is defined by $w = 2 + \mathrm{i}$.

(i) Showing your working, express w^2 in the form $x + \mathrm{i}y$, where x and y are real. Find the modulus of w^2. [3]

(ii) Shade on an Argand diagram the region whose points represent the complex numbers z which satisfy

$$|z - w^2| \leqslant |w^2|.$$ [3]

Q12.

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

Q13.

6 The complex number w is defined by $w = -1 + \mathrm{i}$.

(i) Find the modulus and argument of w^2 and w^3, showing your working. [4]

(ii) The points in an Argand diagram representing w and w^2 are the ends of a diameter of a circle. Find the equation of the circle, giving your answer in the form $|z - (a + b\mathrm{i})| = k$. [4]

Q14.

9 The complex number $1 + (\sqrt{2})\mathrm{i}$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $\mathrm{p}(x)$.

(i) Showing your working, verify that u is a root of the equation $\mathrm{p}(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $\mathrm{p}(x) = 0$. [6]

Q15.

10 (a) Without using a calculator, solve the equation $\mathrm{i}w^2 = (2 - 2\mathrm{i})^2$. [3]

(b) (i) Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z - 4 - 4\mathrm{i}| \leqslant 2.$$ [2]

(ii) For the complex numbers represented by points in the region R, it is given that

$$p \leqslant |z| \leqslant q \quad \text{and} \quad \alpha \leqslant \arg z \leqslant \beta.$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [6]

Q16.

8 **Throughout this question the use of a calculator is not permitted.**

(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\mathrm{i} \quad \text{and} \quad \mathrm{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \mathrm{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

Q17.

9 (a) Without using a calculator, use the formula for the solution of a quadratic equation to solve

$$(2 - \mathrm{i})z^2 + 2z + 2 + \mathrm{i} = 0.$$

Give your answers in the form $a + b\mathrm{i}$. [5]

(b) The complex number w is defined by $w = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$. In an Argand diagram, the points A, B and C represent the complex numbers w, w^3 and w^* respectively (where w^* denotes the complex conjugate of w). Draw the Argand diagram showing the points A, B and C, and calculate the area of triangle ABC. [5]

Q18.

5 The complex number z is defined by $z = \dfrac{9\sqrt{3} + 9\mathrm{i}}{\sqrt{3} - \mathrm{i}}$. Find, showing all your working,

(i) an expression for z in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

(ii) the two square roots of z, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

Q19.

7 (a) The complex number $\frac{3-5\mathrm{i}}{1+4\mathrm{i}}$ is denoted by u. Showing your working, express u in the form $x+\mathrm{i}y$, where x and y are real. [3]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z-2-\mathrm{i}| \leqslant 1$ and $|z-\mathrm{i}| \leqslant |z-2|$. [4]

(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

Q20.

5 **Throughout this question the use of a calculator is not permitted.**

The complex numbers w and z satisfy the relation

$$w = \frac{z+\mathrm{i}}{\mathrm{i}z+2}.$$

(i) Given that $z = 1+\mathrm{i}$, find w, giving your answer in the form $x+\mathrm{i}y$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x+\mathrm{i}y$, where x and y are real. [4]

Q21.

5 The complex numbers w and z are defined by $w = 5+3\mathrm{i}$ and $z = 4+\mathrm{i}$.

(i) Express $\frac{\mathrm{i}w}{z}$ in the form $x+\mathrm{i}y$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]