6 POISSON DISTRIBUTIONS

Objectives

After studying this chapter you should

- be able to recognise when to use the Poisson distribution;
- be able to apply the Poisson distribution to a variety of problems;
- be able to approximate the binomial distribution by a suitable Poisson distribution.

6.0 Introduction

This distribution is introduced through the Activity below.

Activity 1 Vehicle survey

The survey should be carried out on a motorway or dual carriageway, well away from any obstacles that would prevent free flow of traffic such as road works, roundabouts or traffic lights. (A useful, but not essential, piece of apparatus is something that will 'beep' at the end of each minute. Some watches with alarms can be set to go off every minute, or perhaps your chess club has a lightning buzzer. It is something that can be made without much difficulty by an electronics student.)

Note the number of vehicles that pass you in one minute and also the number of lorries. Collect readings for 100 minutes in a period which is either wholly in or completely outside any 'rush hour' surge of traffic that might exist.

For each of your distributions calculate:

(a) mean, $E(X)$ (b) variance, $V(X)$

where X is ' number of vehicles (or lorries) passing in one minute'.

(If several people are involved then the far carriageway can be studied separately. The 'number of red cars' is another example of a variable that could be examined.)

The Poisson distribution,which is developed in the next section, is of particular use when the number of possible occurrences of an event is unlimited. Possible examples are when describing the number of:

(a) flaws in a given length of material;

(b) accidents on a particular stretch of road in a week;

(c) telephone calls made to a switchboard in one day.

6.1 Developing the distribution

Surveys of the type undertaken in Activity 1 are important for transport planners who have to make decisions about road building schemes. For example, in considering the need for an extra lane on a dual carriageway, a survey of the number of lorries over several days gave the number passing a point per minute during the evening rush hour, as in the table opposite.

No. of lorries per minute (x)	Frequency (f)
0	7
1	34
2	84
3	140
4	176
5	176
6	146
7	104
8	65
9	36
10	18
11	8
12	4
13	1
14	1
≥ 15	0

Here is an example of a random variable X, 'the number of lorries per minute', which is certainly going to produce a discrete probability distribution, but each one minute trial will have many possible outcomes.

This cannot be a binomial distribution since, in theory at least, the possible values of X are unlimited.

Now the mean of the data is given by

$$\bar{x} = \frac{0\times7 + 1\times34 + \ldots + 14\times1}{1000} \approx 4.997$$

whilst the variance is

$$s^2 = \frac{0^2\times7 + 1^2\times34 + \ldots + 14^2\times1}{1000} - 4.997^2 \approx 5.013.$$

So allowing a little for experimental error, it seems that the distribution has its mean **equal** to its variance. A relationship between succeeding frequencies can be seen by dividing consecutive data.

$$\frac{34}{7} \approx \frac{5}{1} \qquad \frac{84}{34} \approx \frac{5}{2} \qquad \frac{140}{84} = \frac{5}{3} \qquad \frac{176}{140} \approx \frac{5}{4}$$

$$\frac{176}{176} = \frac{5}{5} \qquad \frac{146}{176} \approx \frac{5}{6} \qquad \frac{104}{146} \approx \frac{5}{7} \qquad \frac{65}{104} = \frac{5}{8}$$

$$\frac{36}{65} \approx \frac{5}{9} \qquad \frac{18}{36} = \frac{5}{10}$$

The initial probability, $P(X=0) = 0.007$, can then be used to calculate the others.

$$P(X=1) = \frac{5}{1}P(X=0)$$

$$P(X=2) = \frac{5}{2}P(X=1) = \frac{5^2}{2\times 1}P(X=0)$$

$$P(X=3) = \frac{5}{3}P(X=2) = \frac{5^3}{3\times 2\times 1}P(X=0)$$

$$P(X=4) = \frac{5^4}{4\times 3\times 2\times 1}P(X=0).$$

Hence the probability distribution can be written

$$P(X=n) = \frac{5^n}{n(n-1)\ \ldots .\ 2\times 1}P(X=0)$$

$$= \frac{5^n}{n!}P(X=0), \quad \text{using factorials.}$$

Since the sum of the probabilities is one, putting $p = P(X=0)$, it follows that

$$1 = p + 5p + \frac{5^2 p}{2!} + \frac{5^3 p}{3!} + \frac{5^4 p}{4!} + \ldots ..$$

$$= p\left(1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \ldots ..\right)$$

$$= pe^5, \quad \text{since } e^5 = 1 + 5 + \frac{5^2}{2!} + \ldots,$$

$$\Rightarrow \quad p = e^{-5}.$$

The exponential number $e \approx 2.71828$ is a very important number in advanced mathematical analysis and can be found on all scientific calculators. The exponential function, e^x, takes the form

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

and it is this result that has been used above with $x=5$.

Activity 2 Telephone calls

Many school and college switchboards are computerised and can provide a print-out of calls over any period. Other switchboards in local firms may also be able to help. Study the number of incoming calls in, for example, ten minute periods, during a time of day avoiding lunch and other breaks. Look at the results for several days. Calculate the mean and variance of your distribution and try to fit a Poisson distribution to your figures.

Activity 3

As an alternative or additional practical to Activity 2, study the number of arrivals of customers at a post office in two minute intervals. The length of the time interval may well be shortened in the case of a large and busy site.

The key parameter in fitting a **Poisson distribution** is the mean value, usually denoted by λ. This is the average number of occurrences in the specified period (e.g. cars passing in a minute). The probability distribution is then given by:

x	0	1	2	3	4	...
$P(X=x)$	$e^{-\lambda}$	$\lambda e^{-\lambda}$	$\dfrac{\lambda^2 e^{-\lambda}}{2!}$	$\dfrac{\lambda^3 e^{-\lambda}}{3!}$	$\dfrac{\lambda^4 e^{-\lambda}}{4!}$	...

In general, if X is a Poisson distribution, then

$$\boxed{P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad (x = 0, 1, 2, \ldots)}$$

and this is denoted by $X \sim Po(\lambda)$.

The **Poisson distribution** was first derived in 1837 by the French mathematician *Simeon Denis Poisson* whose main work was on the mathematical theory of electricity and magnetism.

The distribution arises when the events being counted occur

(a) independently;

(b) such that the probability that two or more events occur simultaneously is zero;

(c) randomly in time or space;

(d) uniformly (that is, the mean number of events in an interval is directly proportional to the length of the interval).

Example

If the random variable X follows a Poisson distribution with mean 3.4, find $P(X=6)$.

Solution

This can be written more quickly as: if $X \sim Po(3.4)$ find $P(X=6)$.

Now $$P(X=6) = \frac{e^{-\lambda}\lambda^6}{6!}$$

$$= \frac{e^{-3.4}(3.4)^6}{6!} \quad (\text{mean, } \lambda = 3.4)$$

$$= 0.071\,604\,409 = 0.072 \quad (\text{to 3 d.p.}).$$

Example

The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5.

Find the probability that

(a) in a particular week there will be:
  (i) less than 2 accidents,
  (ii) more than 2 accidents;

(b) in a three week period there will be no accidents.

Solution

Let A be 'the number of accidents in one week', so $A \sim P_0(0.5)$.

(a) (i) $$P(A<2) = P(A \le 1)$$

$$= 0.9098 \quad (\text{from tables in Appendix 3 (p257), to 4 d.p.})$$

or, from the formula,

$$P(A<2) = P(A=0) + P(A=1)$$

$$= e^{-0.5} + \frac{e^{-0.5} \times 0.5}{1!}$$

$$= \frac{3}{2} e^{-0.5}$$

$$\approx 0.9098.$$

(ii) $P(A > 2) = 1 - P(A \le 2)$

$= 1 - 0.9856$ (from tables)

$= 0.0144$ (to 4 d. p.)

or

$$1 - [P(A=0) + P(A=1) + P(A=2)]$$

$$= 1 - \left[e^{-0.5} + e^{-0.5}0.5 + \frac{e^{-0.5}(0.5)^2}{2!}\right]$$

$$= 1 - e^{-0.5}(1 + 0.5 + 0.125)$$

$$= 1 - 1.625\, e^{-0.5}$$

$$\approx 0.0144.$$

(b) P (0 in 3 weeks) $= \left(e^{-0.5}\right)^3 \approx 0.223.$

Could the number of vehicles on a single carriageway road passing a fixed point in some time interval be expected to follow a Poisson distribution?

Exercise 6A

Give answers to 3 significant figures.

1. If $X \sim Po(3)$, find:

   (a) $P(X=2)$ (b) $P(X=3)$

   (c) $P(X \ge 5)$ (d) $P(X < 3)$.

2. If $X \sim Po(\lambda)$ and $P(X=4) = 3P(X=3)$, find λ and $P(X=5)$.

3. If $X \sim Po(\lambda)$ and $P(X=0) = 0.323$, find the value of λ to two decimal places and use this to calculate $P(X=3)$.

4. Investigate whether or not the following figures might result from a Poisson variable.

x	0	1	2	3	4	5	6	≥ 7
$P(X=x)$	0.368	0.368	0.184	0.061	0.015	0.003	0.001	0.000

*5. If $X \sim Po(2)$, $Y \sim Po(3)$ and $Z \sim Po(5)$, find:

   (a) $P(X+Y=0)$ (b) $P(X+Y=1)$ (c) $P(Z=0)$

   (d) $P(Z=1)$ (e) $P(X+Y \le 2)$ (f) $P(Z \le 2)$

6.2 Combining Poisson variables

Activity 4

The number of telephone calls made by the male and female sections of the P.E. department were noted for fifty days and the results are shown in the table opposite. The number of calls by men are given first in each pair of numbers.

0, 2	2, 2	6, 0	3, 5	1, 2
2, 2	1, 1	2, 2	1, 1	2, 3
7, 0	1, 4	3, 6	2, 3	3, 0
4, 1	5, 1	4, 3	5, 4	6, 4
1, 0	2, 3	3, 2	3, 3	6, 1
2, 3	2, 2	2, 1	3, 5	5, 3
4, 3	4, 2	3, 4	4, 3	3, 1
3, 1	3, 3	4, 4	5, 4	2, 1
5, 6	1, 2	2, 2	1, 2	3, 3
4, 2	0, 5	4, 4	2, 2	4, 1

Number of telephone calls (male, female)

Investigate the distributions of the numbers of calls made:

(a) by the male staff;

(b) by the female staff;

(c) in total each day i.e. $0+2=2$, $2+2=4$, etc.

Activity 5

Study the numbers of cars and lorries from your survey in Activity 1and look at the distributions of the numbers of:

(a) cars (b) lorries (c) all vehicles.

Now consider the result of combining two independent Poisson variables

$$A \sim Po(2) \quad \text{and} \quad B \sim Po(3).$$

Define the new distribution $C = A + B$.

What can you say about C ?

You know that

$$P(A=0) = e^{-2}, \quad P(A=1) = e^{-2} \times 2, \quad P(A=2) = \frac{e^{-2}\,2^2}{2!}, \quad \ldots$$

$$P(B=0) = e^{-3}, \quad P(B=1) = e^{-3} \times 3, \quad P(B=2) = \frac{e^{-3}\,3^2}{2!}, \quad \ldots$$

This gives

$$P(C=0) = P(A=0) \times P(B=0) = e^{-2} \times e^{-3} = e^{-5}.$$

$$
\begin{aligned}
P(C=1) &= P(A=0)\times P(B=1) + P(A=1)\times P(B=0)\\
&= e^{-2}\times 3e^{-3} + 2e^{-2}\times e^{-3} = 5e^{-5},
\end{aligned}
$$

and

$$
\begin{aligned}
P(C=2) &= P(A=0)\times P(B=2) + P(A=1)\times P(B=1)\\
&\quad + P(A=2)\times P(B=0)\\
&= e^{-2}\times\frac{9e^{-3}}{2!} + 2e^{-2}\times 3e^{-3} + e^{-2}\frac{2^2}{2!}\times e^{-3}\\
&= e^{-5}\left(\frac{9}{2!} + 6 + \frac{4}{2!}\right)\\
&= \frac{25e^{-5}}{2!} = \frac{e^{-5}5^2}{2!}.
\end{aligned}
$$

What sort of distribution do these results indicate?

In general, if $A \sim Po(a)$ and $B \sim Po(b)$ are independent random variables, then $C = (A+B) \sim Po(a+b)$.

You have seen this result illustrated in a special case above. The proof requires a good working knowledge of the binomial expansion and is set as an optional activity below.

*Activity 6

By noting that

$$P(C=n) = \sum_{i=0}^{n} P(A=i)\times P(B=n-i)$$

and that

$$(a+b)^n = \sum_{i=0}^{n}\binom{n}{i} a^i b^{n-i}$$

prove that $C \sim Po(a+b)$.

Example

The number of misprints on a page of the *Daily Mercury* has a Poisson distribution with mean 1.2. Find the probability that the number of errors

(a) on page four is 2; (b) on page three is less than 3;

(c) on the first ten pages totals 5;

(d) on all forty pages adds up to at least 3.

Solution

Let E be 'the number of errors on one page', so that
$E \sim Po(1.2)$.

(a) $P(E=2) = \dfrac{e^{-1.2}(1.2)^2}{2!} \approx 0.217$.

(b) $P(E<3) = P(E \le 2) \approx 0.8795$, from tables,

or

$$P(E<3) = P(E=0) + P(E=1) + P(E=2)$$

$$= e^{-1.2} + 1.2e^{-1.2} + \frac{(1.2)^2 e^{-1.2}}{2!}$$

$$= e^{-1.2}(1 + 1.2 + 0.72)$$

$$= 2.92e^{-1.2}$$

$$= 0.8795 \quad \text{(to 4 d.p.)}.$$

(c) Let E_{10} be 'the number of errors on 10 pages',

then $E_{10} \sim Po(12)$, as $E_{10} = E + E + \ldots + E$,

and $E_{10} \sim Po(1.2 + 1.2 + \ldots. + 1.2) = Po(12)$.

Hence $P(E_{10}=5) = \dfrac{e^{-12}12^5}{5!} \approx 0.0127$.

(d) Similarly $E_{40} \sim Po(48)$.

$$P(E_{40} \ge 3) = 1 - P(E_{40} \le 2)$$

$$= 1 - \left(e^{-48} + e^{-48} \times 48 + \frac{e^{-48} \times 48^2}{2!}\right)$$

$$= 1 - 1201e^{-48} \approx 1.000 \text{ (to 3 d.p.)}.$$

Tables may well be a time saving device, as was true with the binomial distribution.

Activity 7

A firm has three telephone numbers. They all receive numbers of calls that follow Poisson distributions, the first having a mean of 8, the second 4 and the third 3 in a period of half an hour. Find the probability that

(a) the second and third lines will receive a total of exactly six calls in half an hour;

(b) the firm will receive at least twelve calls in half an hour;

(c) line two will receive at most six calls in one hour;

(d) line one will receive no calls in 15 minutes.

Example

A shop sells a particular make of video recorder.

(a) Assuming that the weekly demand for the video recorder is a Poisson variable with mean 3, find the probability that the shop sells

(i) at least 3 in a week,

(ii) at most 7 in a week,

(iii) more than 20 in a month (4 weeks).

Stocks are replenished only at the beginning of each month.

(b) Find the minimum number that should be in stock at the beginning of a month so that the shop can be at least 95% sure of being able to meet the demands during the month.

Solution

(a) Let X be the demand in a particular week. Thus $X \sim Po(3)$ and, using the Poisson tables in the Appendix,

(i) $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.4232$ (Note that the tables give the
$= 0.5768$ cumulative probabilities)

(ii) $P(X \leq 7) = 0.9881$, from tables.

(iii) If Y denotes the demand in a particular month, then

$Y \sim Po(12)$ and

$$\begin{aligned} P(Y > 20) &= 1 - P(Y \leq 20) \\ &= 1 - 0.9884 \\ &= 0.0116. \end{aligned}$$

(b) You need to find the smallest value of n such that

$$P(Y \le n) \ge 0.95.$$

From the tables,

$$P(Y \le 17) = 0.9370, \quad P(Y \le 18) = 0.9626$$

So the required minimum stock is 18.

A very important property of the Poisson distribution is that if $X \sim Po(\lambda)$, then

$$E(X) = V(X) = \lambda.$$

That is, both the mean and variance of a Poisson distribution are equal to λ.

To show that $E(X) = \lambda$, note that, by definition,

$$\begin{aligned} E(X) &= \sum_{\text{all } x} xP(X = x) \\ &= 0 \times e^{-\lambda} + 1 \times \left(\lambda e^{-\lambda}\right) + 2 \times \left(\frac{\lambda^2 e^{-\lambda}}{2!}\right) + 3 \times \left(\frac{\lambda^3 e^{-\lambda}}{3!}\right) + \ldots \\ &= \lambda e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\right) \\ &= \lambda e^{-\lambda} e^{\lambda} \quad \text{(using the result from page 117)} \\ &= \lambda \end{aligned}$$

which proves the result. The proof of $V(X) = \lambda$ follows in a similar but more complicated way.

*Activity 8

If $X \sim Po(\lambda)$ show that $V(X) = \lambda$.

Exercise 6B

1. Incoming telephone calls to a school arrive at random times. The average rate will vary according to the day of the week. On Monday mornings, in term time there is a constant average rate of 4 per hour. What is the probability of receiving

   (a) 6 or more calls in a particular hour,

   (b) 3 or fewer calls in a particular period of two hours?

   During term time on Friday afternoons the average rate is also constant and it is observed that the probability of no calls being received during a particular hour is 0.202. What is the average rate of calls on Friday afternoons? (AEB)

2. Write down two conditions which need to be satisfied in order to use the Poisson distribution.

   The demand at a garage for replacement windscreens occurs randomly and at an average rate of 5 per week.

   Determine the probability that no more than 7 windscreens are demanded in a week.

   The windscreen manufacturer uses glass which contains random flaws at an average rate of 48 per 100 m^2. A windscreen of area 0.95 m^2 is chosen at random.

   Determine the probability that the windscreen has fewer than 2 flaws.

   A random sample of 5 such windscreens is taken. Find the probability that exactly 3 of them contain fewer than 2 flaws. (AEB)

3. A garage uses a particular spare part at an average rate of 5 per week. Assuming that usage of this spare part follows a Poisson distribution, find the probability that

   (a) exactly 5 are used in a particular week,

   (b) at least 5 are used in a particular week,

   (c) exactly 15 are used in a 3-week period,

   (d) at least 15 are used in a 3-week period,

   (e) exactly 5 are used in each of 3 successive weeks.

   If stocks are replenished weekly, determine the number of spare parts which should be in stock at the beginning of each week to ensure that on average the stock will be insufficient on no more than one week in a 52 week year. (AEB)

4. A shopkeeper hires vacuum cleaners to the general public at £5 per day. The mean daily demand is 2.6.

   (a) Calculate the expected daily income from this activity assuming an unlimited number of vacuum cleaners is available.

   The demand follows a Poisson distribution.

   (b) Find the probability that the demand on a particular day is:

   (i) 0 (ii) exactly one

   (iii) exactly two (iv) three or more.

   (c) If only 3 vacuum cleaners are available for hire calculate the mean of the daily income.

   A nearby large store is willing to lend vacuum cleaners at short notice to the shopkeeper, so that in practice she will always be able to meet demand. The store would charge £2 per day for this service regardless of how many, if any, cleaners are actually borrowed. Would you advise the shopkeeper to take up this offer? Explain your answer. (AEB)

6.3 The Poisson distribution as an approximation to the binomial

Despite having tables and powerful calculators, it is often difficult to make binomial calculations if *n*, the number of experiments, becomes very large. In these circumstances, it is easier to approximate the binomial by a Poisson distribution.

Activity 9 Birthday dates

Obtain the dates of birth of the students in your college or school from official records or by running a survey. Record the number of people having a birthday on each date (omitting 29th February). Draw up a frequency table.

Use the theoretical probability $\frac{1}{365}$ to work out the expected frequencies of 0, 1, 2, ... people from the *n* people considered, who have birthdays on the same date, according to the binomial distribution $B\left(n, \frac{1}{365}\right)$. Using a mean number of people per day, $\frac{n}{365}$, calculate the expected frequencies from the Poisson distribution $Po\left(\frac{n}{365}\right)$.

Compare the observed and two sets of expected frequencies.

Poisson's first work on the distribution that bears his name arose from considering the binomial distribution and it was derived as an approximation to the already known binomial model.

When might a Poisson distribution give probabilities close to those of a binomial distribution?

If $X \sim B(n, p) \approx Po(\lambda)$, then the means and variances of the two distributions must be about the same. This gives

mean : $np = \lambda$

variance : $npq = \lambda$.

So, if p is small (for example $p = \frac{1}{365}$ in the Activity above), then

$$q \approx 1 \quad \text{and} \quad n = \frac{\lambda}{p}.$$

So when n is very large and p is very small, binomial probabilities may be approximated by Poisson probabilities with $\lambda = np$.

Normally, for this approximation it is required that

$$\boxed{n \geq 50 \quad \text{and} \quad p \leq 0.1}$$

The approximation improves as $n \to \infty$, $p \to 0$.

Example

A factory produces nails and packs them in boxes of 200. If the probability that a nail is substandard is 0.006, find the probability that a box selected at random contains at most two nails which are substandard.

Solution

If X is 'the number of substandard nails in a box of 200', then

$$X \sim B(200,\ 0.006).$$

Since n is large and p is small, the Poisson approximation can be used. The appropriate value of λ is given by

$$\lambda = np = 200 \times 0.006 = 1.2.$$

So $\quad X \sim Po(1.2)$,

and $\quad P(X \le 2) = 0.8795$ (from tables),

or
$$P(X \le 2) = e^{-1.2} + e^{-1.2} \times 1.2 + \frac{e^{-1.2}1.2^2}{2!}$$
$$= 2.92e^{-1.2}$$
$$= 0.8795 \quad \text{(to 4 d.p.).}$$

One of the advantages of the use of a Poisson approximation is that tables can be used more often to avoid routine calculations.

Exercise 6C

Where appropriate, give answers to 3 significant figures.

1. If $X \sim B(500,\ 0.002)$, use the binomial and Poisson distributions to find:

   (a) $P(X=0)$ (b) $P(X=1)$ (c) $P(X=4)$.

2. If $X \sim B(200,\ 0.06)$, use Poisson tables to find the values of:

   (a) $P(X<20)$ (b) $P(X \ge 5)$.

3. Fuses are packed in boxes of 1000. If 0.2% are faulty find the probability that a box will contain

   (a) exactly 2 faulty; (b) at least one faulty.

4. A link in a metal chain has probability 0.03 of breaking under a load of 50 kg. What is the probability that a chain made of 100 such links will break when subjected to a 50 kg load?

5. The number of runs scored by Ali in an innings of a cricket match is distributed according to a Poisson distribution with mean 4.5. Find the probability that he will score:

   (a) exactly 4 in his next innings;

   (b) at least three in his next innings;

   (c) at least six in total in his next two innings.

6. State two conditions under which a binomial distribution may be approximated by a Poisson distribution, and give a reason why this approximation may be useful in practice.

   In the treatment of hay fever, the probability that any sufferer is allergic to a particular drug is 0.0005. Assuming that the occurrences of the allergy in different sufferers are independent, find the probability that in a random sample of 8000 sufferers more than four will be allergic to the drug.

   Each sufferer who is allergic to the drug has a probability of 0.3 of developing serious complications following its administration.

   (a) Determine the probability that, of the 8000 sufferers who are administered the drug, exactly two develop serious complications.

   In fact four sufferers develop the allergy following the administration of the drug to the random sample of 8000 sufferers.

   (b) Determine the probability that exactly two of these four develop serious complications.

   Explain, briefly, why your answers to (a) and (b) differ. (AEB)

6.4 Miscellaneous Exercises

1. The number of goals scored in a hockey match by Sarindar and Paula are independent Poisson variables with means 2.5 and 1.5 respectively. Find the probabilities that in a particular match:

   (a) Sarindar will score at least twice;

   (b) they will score at most three between them.

2. If 8% of a city is affected by an outbreak of flu, use the Poisson approximation to the binomial distribution to find the probability that a factory with 160 employees will have at least five people absent.

3. If X is a random variable which has a Poisson distribution with mean 3.99, what is the most likely value of X?

   Explain your result in terms of the relationship between frequencies.

4. A van hire firm has twelve vehicles available and has found that demand follows a Poisson distribution with mean 9.5. In a month of 25 working days, on how many days would you expect:

   (a) demand to exceed supply;

   (b) all vehicles to be idle;

   (c) it to be possible to service 3 of the vans?

5. The number of errors made by a typist on a single page is a Poisson variable with mean 0.09. Find the probability that a fifty page article will have:

   (a) at least 3 errors;

   (b) no errors on the first ten pages.

6. Use the Poisson approximation to the binomial distribution to calculate the probability that a consignment of 10 000 electronic components, each of which has a 0.02% probability of being faulty, contains only perfect items.

   If eight consignments are received, what is the most likely number to contain no faulty components?

7. What is the probability that a Poisson variable with mean 5 will produce exactly two 3's in four trials?

8. A newsagent finds that the mean number of copies of a particular magazine he sells each week is 10. If the number sold follows a Poisson distribution, find the probability that he sells less than four in a week.

   How many should he have in stock at the start of the week if the chance that he cannot provide a customer with a copy is less than 0.05?

9. Take any English novel and use random numbers to select a page and then a line on a page. Starting at the beginning of the line, count 50 letters and note the number of occurrences of double letters.

   Repeat the process until you have at least fifty results. Compare your figures with those from a Poisson distribution.

10. The number of bacteria in one millilitre of a liquid is known to follow a Poisson distribution with mean 3. Find the probability that a 1 ml sample will contain no bacteria. If 100 samples are taken, find the probability that at most ten will contain no bacteria. (Use a Poisson approximation and give your answer to the first part correct to 3 d.p.)

11. The numbers in a group booking into a hotel are found to follow a Poisson distribution with mean 2.2. What is the probability that the next booking will be for a party of more than three?

    What is the probability that just one of the next four bookings will be for such a group?

12. The number of parasites on fish hatched in the same season and living in the same pond follows a Poisson distribution with mean 3.6.

    Find, giving your answers to 3 decimal places, the probability that a fish selected at random will have

    (a) 4 or less parasites, (b) exactly 2 parasites.

    (AEB)

13. Customers at a motorway service station enter the cafeteria through a turnstile. The cafeteria is open 24 hours a day and an automatic counting device records the number of people entering each minute. State, giving reasons, whether or not it is likely that these data will follow a Poisson distribution. (AEB)

14. In the manufacture of commercial carpet, small faults occur at random in the carpet at an average rate of 0.95 per 20 m^2.

    Find the probability that in a randomly selected 20 m^2 area of this carpet

    (a) there are no faults (b) there at most 2 faults.

    The ground floor of a new office block has 10 rooms. Each room has an area of 80 m^2 and has been carpeted using the same commercial carpet described above.

    For any one of these rooms, determine the probability that the carpet in that room

    (c) contains at least 2 faults,

    (d) contains exactly 3 faults,

    (e) contains at most 5 faults.

    Find the probability that in exactly half of these 10 rooms the carpets will contain exactly 3 faults.

    (AEB)

15. A polytechnic offers a short course on advanced statistical methods. As the course involves a large amount of practical work only 8 places are available. Advertising starts two months before the course and if, at the end of one month, 3 or fewer places have been taken the course is cancelled. If 4 or more places have been taken by the end of one month the course is run regardless of the number of applications received in the second month. If the number of applications per month follows a Poisson distribution with mean 3.6, and places are allocated on a first come first served basis, what is the probability that at the end of one month the course will be

(a) cancelled,

(b) full?

What is the probability that

(c) a place will be available at the start of the second month,

(d) the course will run with 8 students?

If the course is offered on four separate occasions, what is the probability that it will

(e) run once and be cancelled three times,

(f) run with 8 students on 2 or more occasions?

(AEB)