Chapter 4 Differentiation

May/June 2002

5 The equation of a curve is $y = 2\cos x + \sin 2x$. Find the x-coordinates of the stationary points on the curve for which $0 < x < \pi$, and determine the nature of each of these stationary points. [7]

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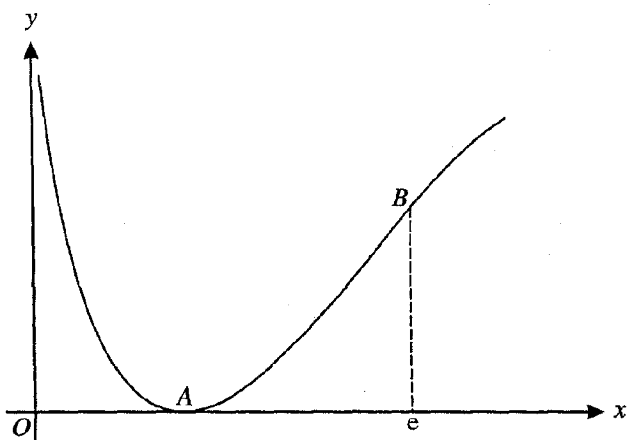

The function f is defined by $f(x) = (\ln x)^2$ for $x > 0$. The diagram shows a sketch of the graph of $y = f(x)$. The minimum point of the graph is A. The point B has x-coordinate e.

(i) State the x-coordinate of A. [1]

(ii) Show that $f''(x) = 0$ at B. [4]

(iii) Use the substitution $x = e^u$ to show that the area of the region bounded by the x-axis, the line $x = e$, and the part of the curve between A and B is given by

$$\int_0^1 u^2 e^u \, du.$$ [3]

(iv) Hence, or otherwise, find the exact value of this area. [3]

Oct/Nov 2002

4 The curve $y = e^x + 4e^{-2x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether the stationary point is a maximum or a minimum point. [2]

May/June 2003

8 The equation of a curve is $y = \ln x + \dfrac{2}{x}$, where $x > 0$.

(i) Find the coordinates of the stationary point of the curve and determine whether it is a maximum or a minimum point. [5]

(ii) The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2}{3 - \ln x_n},$$

with initial value $x_1 = 1$, converges to α. State an equation satisfied by α, and hence show that α is the x-coordinate of a point on the curve where $y = 3$. [2]

(iii) Use this iterative formula to find α correct to 2 decimal places, showing the result of each iteration. [3]

Oct/Nov 2003

4 The equation of a curve is

$$\sqrt{x} + \sqrt{y} = \sqrt{a},$$

where a is a positive constant.

(i) Express $\dfrac{dy}{dx}$ in terms of x and y. [3]

(ii) The straight line with equation $y = x$ intersects the curve at the point P. Find the equation of the tangent to the curve at P. [3]

May/June 2004

3 Find the gradient of the curve with equation

$$2x^2 - 4xy + 3y^2 = 3,$$

at the point (2, 1). [4]

Oct/Nov 2004

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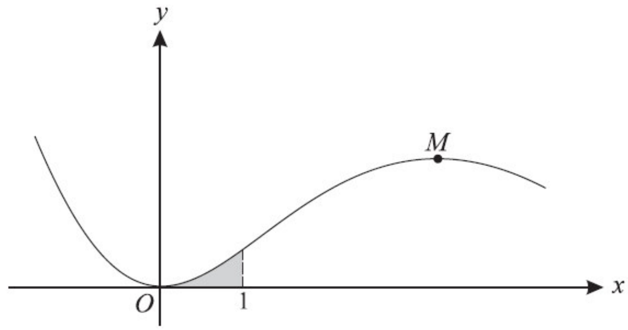

The diagram shows the curve $y = x^2 e^{-\frac{1}{2}x}$.

(i) Find the x-coordinate of M, the maximum point of the curve. [4]

(ii) Find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 1$, giving your answer in terms of e. [5]

Oct/Nov 2005

3 The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

May/June 2006

3 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 1 - \cos 2\theta.$$

Show that $\frac{dy}{dx} = \tan\theta$. [5]

Oct/Nov 2006

3 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

6 The equation of a curve is $x^3 + 2y^3 = 3xy$.

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{2y^2 - x}$. [4]

(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

May/June 2007

3 The equation of a curve is $y = x \sin 2x$, where x is in radians. Find the equation of the tangent to the curve at the point where $x = \frac{1}{4}\pi$. [4]

Oct/Nov 2007

4 The curve with equation $y = e^{-x} \sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

May/June 2008

6 The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

Oct/Nov 2008

3 The curve $y = \dfrac{e^x}{\cos x}$, for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, has one stationary point. Find the x-coordinate of this point. [5]

4 The parametric equations of a curve are

$$x = a(2\theta - \sin 2\theta), \qquad y = a(1 - \cos 2\theta).$$

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

May/June 2009

6 The parametric equations of a curve are

$$x = a\cos^3 t, \quad y = a\sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin t + y\cos t = a\sin t\cos t. \qquad [3]$$

(iii) Hence show that, if this tangent meets the x-axis at X and the y-axis at Y, then the length of XY is always equal to a. [2]

Oct/Nov 2009/31

4 A curve has equation $y = e^{-3x}\tan x$. Find the x-coordinates of the stationary points on the curve in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. Give your answers correct to 3 decimal places. [6]

Oct/Nov 2009/32

3 The equation of a curve is $x^3 - x^2y - y^3 = 3$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 1), giving your answer in the form $ax + by + c = 0$. [2]

May/June 2010/31

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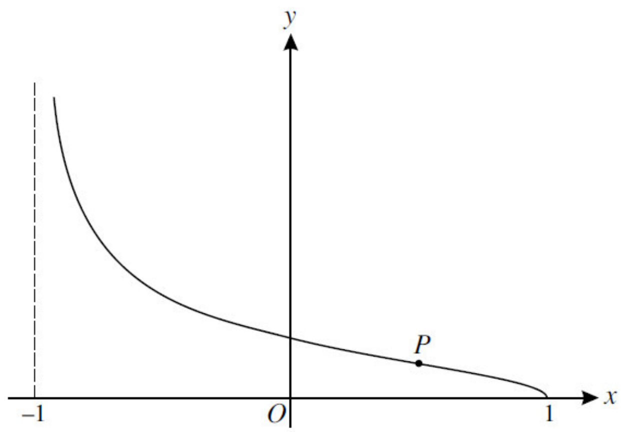

The diagram shows the curve $y = \sqrt{\left(\frac{1-x}{1+x}\right)}$.

(i) By first differentiating $\frac{1-x}{1+x}$, obtain an expression for $\frac{dy}{dx}$ in terms of x. Hence show that the gradient of the normal to the curve at the point (x, y) is $(1+x)\sqrt{(1-x^2)}$. [5]

(ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x-coordinate of P. [4]