Chapter 1 Algebra

May/June 2002

2 Expand $(1 - 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 The polynomial $x^4 + 4x^2 + x + a$ is denoted by $p(x)$. It is given that $(x^2 + x + 2)$ is a factor of $p(x)$.

Find the value of a and the other quadratic factor of $p(x)$. [4]

6 Let $f(x) = \dfrac{4x}{(3x+1)(x+1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = 1 - \ln 2$. [5]

Oct/Nov 2002

1 Solve the inequality $|9 - 2x| < 1$. [3]

6 Let $f(x) = \dfrac{6+7x}{(2-x)(1+x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = 3 + 5x - \tfrac{1}{2}x^2 - \tfrac{15}{4}x^3.$$ [5]

May/June 2003

3 Solve the inequality $|x - 2| < 3 - 2x$. [4]

4 The polynomial $x^4 - 2x^3 - 2x^2 + a$ is denoted by $f(x)$. It is given that $f(x)$ is divisible by $x^2 - 4x + 4$.

(i) Find the value of a. [3]

(ii) When a has this value, show that $f(x)$ is never negative. [4]

6 Let $f(x) = \dfrac{9x^2+4}{(2x+1)(x-2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that, when x is sufficiently small for x^3 and higher powers to be neglected,

$$f(x) = 1 - x + 5x^2.$$ [4]

Oct/Nov 2003

1 Solve the inequality $|2^x - 8| < 5$. [4]

2 Expand $(2 + x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

8 Let $\mathrm{f}(x) = \dfrac{x^3 - x - 2}{(x-1)(x^2+1)}$.

(i) Express $\mathrm{f}(x)$ in the form

$$A + \frac{B}{x-1} + \frac{Cx + D}{x^2+1},$$

where A, B, C and D are constants. [5]

(ii) Hence show that $\int_2^3 \mathrm{f}(x)\,\mathrm{d}x = 1$. [4]

May/June 2004

2 Solve the inequality $|2x + 1| < |x|$. [4]

9 Let $\mathrm{f}(x) = \dfrac{x^2 + 7x - 6}{(x-1)(x-2)(x+1)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$\mathrm{f}(x) = -3 + 2x - \tfrac{3}{2}x^2 + \tfrac{11}{4}x^3.$$ [5]

Oct/Nov 2004

1 Expand $\dfrac{1}{(2+x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

3 The polynomial $2x^3 + ax^2 - 4$ is denoted by $\mathrm{p}(x)$. It is given that $(x - 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $\mathrm{p}(x)$, [2]

(iii) solve the inequality $\mathrm{p}(x) > 0$, justifying your answer. [2]

8 An appropriate form for expressing $\dfrac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1}+\frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\dfrac{4x}{(x+4)(x^2+3)}$, [1]

(ii) $\dfrac{2x+1}{(x-2)(x+2)^2}$. [2]

(b) Show that $\displaystyle\int_3^4 \frac{3x}{(x+1)(x-2)}\,dx = \ln 5$. [6]

May/June 2005

1 Expand $(1+4x)^{-\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

5 The polynomial x^4+5x+a is denoted by $p(x)$. It is given that x^2-x+3 is a factor of $p(x)$.

(i) Find the value of a and factorise $p(x)$ completely. [6]

(ii) Hence state the number of real roots of the equation $p(x)=0$, justifying your answer. [2]

Oct/Nov 2005

1 Given that a is a positive constant, solve the inequality

$$|x-3a| > |x-a|.$$ [4]

9 **(i)** Express $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2+x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

May/June 2006

2 Solve the inequality $2x > |x-1|$. [4]

9 **(i)** Express $\dfrac{10}{(2-x)(1+x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\dfrac{10}{(2-x)(1+x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

Oct/Nov 2006

1 Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer. [4]

5 **(i)** Simplify $(\sqrt{(1+x)} + \sqrt{(1-x)})(\sqrt{(1+x)} - \sqrt{(1-x)})$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

8 Let $\mathrm{f}(x) = \dfrac{7x+4}{(2x+1)(x+1)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = 2 + \ln\tfrac{5}{3}$. [5]

May/June 2007

1 Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, find the quadratic factor of $\mathrm{p}(x)$. [2]

Oct/Nov 2007

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $x^2 + x + 2$ is a factor of $\mathrm{p}(x)$. Find the value of a and the other quadratic factor of $\mathrm{p}(x)$. [4]

9 (i) Express $\frac{2 - x + 8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2 - x + 8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x, up to and including the term in x^2. [5]

May/June 2008

1 Solve the inequality $|x - 2| > 3|2x + 1|$. [4]

7 Let $f(x) \equiv \frac{x^2 + 3x + 3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^3 f(x)\,dx = 3 - \frac{1}{2}\ln 2$. [4]

Oct/Nov 2008

2 Expand $(1 + x)\sqrt{(1 - 2x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

5 The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a. [3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer. [3]

May/June 2009

5 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

8 (i) Express $\frac{100}{x^2(10 - x)}$ in partial fractions. [4]

(ii) Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{100}x^2(10 - x),$$

obtaining an expression for t in terms of x. [6]

Oct/Nov 2009/31

1 Solve the inequality $2 - 3x < |x - 3|$. [4]

8 **(i)** Express $\frac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{5x+3}{(x+1)^2(3x+2)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

Oct/Nov 2009/32

5 The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $\mathrm{p}(x)$. The result of differentiating $\mathrm{p}(x)$ with respect to x is denoted by $\mathrm{p}'(x)$. It is given that $(x+2)$ is a factor of $\mathrm{p}(x)$ and of $\mathrm{p}'(x)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

8 **(i)** Express $\frac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1+x}{(1-x)(2+x^2)}$ in ascending powers of x, up to and including the term in x^2. [5]

May/June 2010/31

1 Solve the inequality $|x+3a| > 2|x-2a|$, where a is a positive constant. [4]

8 **(i)** Express $\frac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part **(i)**, show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

(iii) Hence show that $\int_0^1 \frac{4}{(x+1)^2(x+3)^2}\,dx = \frac{7}{12} - \ln\frac{3}{2}$. [5]

May/June 2010/32

5 The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x+1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x+2)$ the remainder is 9.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

10 **(i)** Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x-1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x-1}.$$ [5]

(ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}\,dx = \frac{3}{2} + \frac{1}{2}\ln\left(\frac{16}{27}\right).$$ [5]

May/June 2010/33

1 Solve the inequality $|x - 3| > 2|x + 1|$. [4]

9 **(i)** Express $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]