SPM ADDITIONAL MATHEMATIC 3472

CHAPTER 9 : DIFFERENTIATIONS

Topic 9 Differentiation :::::::::: v 9.01 Definitions & First Principles Differentiation ::: v 9.011 Statement Definitions *[Takrif Ayat]*

When x increases by $+\delta y$ δx, y will increases by δy.

e.g. $y = x$

$y + \delta y = x + \delta x$

$\delta x \Rightarrow$ small change of x
$\delta y \Rightarrow$ small change of y
$\delta \Rightarrow DELTA$

Topic 9 Differentiation :::::::::: v 9.01 Definitions & First Principles Differentiation ::: v 9.012 First Principles Differentiation (Use the definition) *[Pembezaan Prinsip Pertama]*

e.g.1:

Find the first derivative of $y = x^2$ from first principles.

Solution:

$\frac{dy}{dx} \Rightarrow$ "dee y dee x"

$y = x^2$ .... {1}

y increases by δy → $y + \delta y = (x + \delta x)^2$ ← x increases by δx

$y + \delta y = x^2 + 2\delta x \bullet x + (\delta x)^2$ .... {2}

{2} − {1},

$(y + \delta y) - y = (x^2 + 2\delta x \bullet x + (\delta x)^2) - x^2$

$\delta y = 2\delta x \bullet x + (\delta x)^2$

$(\div \delta x)$

$\frac{\delta y}{\delta x} = 2x + \delta x$

The symbol of definition for differentiation → $\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) = 2x + 0$

shortcut of symbol for differentiation → $\frac{dy}{dx} = 2x$ #

Important Notes:

$\frac{dy}{dx} \neq \frac{\delta y}{\delta x}$

$\frac{dy}{dx} = \lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right)$

e.g.2:

Find the first derivative of $y = 3x + 2$ from first principles.

Solution:

$y = 3x + 2$ .... {1}

$y + \delta y = 3(x + \delta x) + 2$

$y + \delta y = 3x + 3\delta x + 2$ .... {2}

{2} − {1},

$(y + \delta y) - y = (3x + 3\delta x + 2) - (3x + 2)$

$\delta y = 3\delta x$

$(\div \delta x)$

$\frac{\delta y}{\delta x} = 3$

$\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) = 3$

$\frac{dy}{dx} = 3$ #

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e.g.3:

Find the first derivative of $y = \frac{1}{x}$ from first principles.

Solution:

$$y = \frac{1}{x} \ldots\ldots\ldots \{1\}$$

$$y + \delta y = \frac{1}{x + \delta x} \ldots\ldots\ldots \{2\}$$

$\{2\} - \{1\}$,

$$(y + \delta y) - y = \left(\frac{1}{x + \delta x}\right) - \left(\frac{1}{x}\right)$$

$$\delta y = \left(\frac{1}{x + \delta x}\right) - \left(\frac{1}{x}\right)$$

$$\delta y = \left(\frac{x - (x + \delta x)}{x(x + \delta x)}\right)$$

$$\delta y = \left(\frac{-\delta x}{x^2 + x(\delta x)}\right)$$

$$(\div \delta x)$$

$$\frac{\delta y}{\delta x} = \left(\frac{-1}{x^2 + x(\delta x)}\right)$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \left(\frac{-1}{x^2 + x(0)}\right)$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \left(-\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = -\frac{1}{x^2} \#$$

---

e.g.4:

Find the first derivative of $y = \frac{2}{x+1}$ from first principles.

Solution:

$$y = \frac{2}{x+1} \ldots\ldots\ldots \{1\}$$

$$y + \delta y = \frac{2}{(x + \delta x) + 1} \ldots\ldots\ldots \{2\}$$

$\{2\} - \{1\}$,

$$(y + \delta y) - y = \left(\frac{2}{x + \delta x + 1}\right) - \left(\frac{2}{x+1}\right)$$

$$\delta y = \left(\frac{2(x+1) - 2(x + \delta x + 1)}{(x+1)(x + \delta x + 1)}\right)$$

$$\delta y = \left(\frac{(2x+2) - (2x + 2(\delta x) + 2)}{x^2 + x(\delta x) + x + x + (\delta x) + 1}\right)$$

$$\delta y = \left(\frac{-2(\delta x)}{x^2 + x(\delta x) + 2x + (\delta x) + 1}\right)$$

$$(\div \delta x)$$

$$\frac{\delta y}{\delta x} = \left(\frac{-2}{x^2 + x(\delta x) + 2x + (\delta x) + 1}\right)$$

$$\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right) = \left(\frac{-2}{x^2 + x(0) + 2x + (0) + 1}\right)$$

$$= \left(\frac{-2}{x^2 + 2x + 1}\right)$$

$$\frac{dy}{dx} = -\frac{2}{(x+1)^2} \#$$

Topic 9 Differentiation :::::::::: v 9.02 4 Formulae Of Differentiation / *Pembezaan 4 Rumus* /::::: v 9.021 Basic Formulae Of Differentiation / *Pembezaan Rumus Asas* /

$$y = x^n$$

$$\frac{dy}{dx} = nx^{n-1}$$

e.g.1:

Differentiate $y = 3x^5$ with respect to x.

Solution:

$y = 3x^5$

$\frac{dy}{dx} = 5 \times 3x^{5-1}$

$\frac{dy}{dx} = 15x^4$ #

***e.g.2:[memories]

Differentiate $y = 3x$ with respect to x.

Solution:

$y = 3x$

$\frac{dy}{dx} = 1 \times 3x^{1-1}$

$\frac{dy}{dx} = 1 \times 3x^0$

$\frac{dy}{dx} = 3$#

***e.g.3:[memories]

Differentiate $y = 2$ with respect to x.

Solution:

$y = 2x^0$

$\frac{dy}{dx} = 0 \times 2x^{0-1}$

$\frac{dy}{dx} = 0$ #

e.g.4:

Differentiate $y = x^2$ with respect to x.

Solution:

$y = x^2$

$\frac{dy}{dx} = 2 \times x^{2-1}$

$\frac{dy}{dx} = 2x$

$\frac{d}{dx}(y) = 2x$

$\frac{d}{dx}(x^2) = 2x$#

e.g.5:

Differentiate $x^3 - 2x$ with respect to x.

Solution:

$\frac{d}{dx}(x^3 - 2x) = 3x^{3-1} - 2x^{1-1}$

$= 3x^2 - 2$#

e.g.6:

$y = 3x^6$.

Solution:

$\frac{dy}{dx} = 6 \times 3x^{6-1} = 18x^5$

$\frac{d^2y}{dx^2} = 5 \times 18x^{5-1} = 90x^4$

$\frac{d^3y}{dx^3} = 4 \times 90x^{4-1} = 360x^3$

$f(x) = 3x^6$.

Solution:

$f'(x) = 6 \times 3x^{6-1} = 18x^5$

$f''(x) = 5 \times 18x^{5-1} = 90x^4$

$f'''(x) = 4 \times 90x^{4-1} = 360x^3$

e.g.7:

Differentiate $y = \frac{1}{x}$ with respect to x..

Solution:

$y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -1 \times x^{-1-1}$

$\frac{dy}{dx} = -1 \times x^{-2}$

$\frac{dy}{dx} = -\frac{1}{x^2}$ #

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Topic 9 Differentiation :::::::::: v 9.02 4 Formulae Of Differentiation :::::: v 9.022 Brackets Rule *(Rumus Kurungan)*

[**Some "Brackets" differentiation can be avoided]

e.g.1:

Differentiate $y = 3\left(x^2+2\right)^4$ with respect to x.

Solution:

$y = px^n$

$\frac{dy}{dx} = npx^{n-1}\left(\textit{differentiate inside}\right)$

$$y = 3\left(x^2+2\right)^4$$

$$\frac{dy}{dx} = 4\times 3\left(x^2+2\right)^{4-1}\boxed{(2x)} \rightarrow \textit{differentiate the "brackets"}$$

$$\frac{dy}{dx} = 12(2x)\left(x^2+2\right)^3$$

$$\frac{dy}{dx} = 24x\left(x^2+2\right)^3 \ \#$$

e.g.2:

Differentiate $y = \left(2x+\frac{1}{x}\right)^2$ with respect to x.

Solution:

*expands first before we differentiate it

$$y = \left(2x+\frac{1}{x}\right)^2$$

$$y = 4x^2+2(2x)\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2$$

$$y = 4x^2+4+\left(\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = 8x-\frac{2}{x^{3^2}} \ \#$$

e.g.3:

Differentiate $y=\dfrac{3}{2x-1}$ with respect to x.

Solution:

$$y=\frac{3}{2x-1}$$
$$y=3\left(\frac{1}{2x-1}\right)$$
$$y=3(2x-1)^{-1}$$
$$\frac{dy}{dx}=-1\times3(2x-1)^{-1-1}(2)$$
$$\frac{dy}{dx}=-6(2x-1)^{-2}$$
$$\frac{dy}{dx}=-\frac{6}{(2x-1)^2}\ \#$$

e.g.4:

Differentiate $I=\dfrac{1}{3}\pi r\left(r^2-\dfrac{1}{r^2}\right)$ with respect to r.

Solution:

$$I=\frac{1}{3}\pi \boxed{r}\left(r^2-\frac{1}{r^2}\right)$$
$$I=\frac{1}{3}\pi\left(r^3-\frac{1}{r}\right)$$
$$\frac{dI}{dr}=\frac{1}{3}\pi\left(3r^{3-1}-(-1)\left(r^{-1-1}\right)\right)$$
$$\frac{dI}{dr}=\boxed{\frac{1}{3}\pi}\left(3r^2+\frac{1}{r^2}\right)\#$$

The constants no need involved in defferentiation.

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Topic 9 Differentiation :::::::::: v 9.02 4 Formulae Of Differentiation ::::: v 9.023 Product Rule (*Pembezaan Pendaraban*) :

[Can be avoided]

$$y=\boxed{1}\times\boxed{2}$$
$$\frac{dy}{dx}=Differentiate\boxed{1}\times keep\boxed{2}+Differentiate\boxed{2}\times keep\boxed{1}$$
$$y=uv,\ \frac{dy}{dx}=\frac{du}{dx}v+\frac{dv}{dx}u$$

e.g.1:

Differentiate $y=x^2(2x+1)^4$ with respect to x.

Solution:

$$y=x^2(2x+1)^4$$

$\boxed{1}$ $\boxed{2}$

$$\frac{dy}{dx}=2x(2x+1)^4+4(2x+1)^3(2)x^2$$
$$\frac{dy}{dx}=2x(2x+1)^4+8x^2(2x+1)^3$$
$$\frac{dy}{dx}=2x(2x+1)^3[2x+1+4x]$$
$$\frac{dy}{dx}=2x(2x+1)^3(6x+1)\#$$

e.g.2:

Differentiate $y=2x(x^2-1)^3$ with respect to x.

Solution:

$$y=2x(x^2-1)^3$$
$$\frac{dy}{dx}=2(x^2-1)^3+3(x^2-1)^2(2x)2x$$
$$\frac{dy}{dx}=2(x^2-1)^3+12x^2(x^2-1)^2$$
$$\frac{dy}{dx}=2(x^2-1)^2[(x^2-1)+6x^2]$$
$$\frac{dy}{dx}=2(x^2-1)^2(7x^2-1)\#$$

e.g.3:

Differentiate $y = x(x+1)^2$ with respect to x.

Solution:

$y = x(x+1)^2$

$y = x(x^2+2x+1)$

$y = x^3+2x^2+x$

$\frac{dy}{dx} = 3x^2+4x+1$ #

Topic 9 Differentiation :::::::::: v 9.02 4 Formulae Of Differentiation :::::: v 9.024 Quotient Rule (*Pembezaan Pembahagian*) :
[Can be avoided]

$$y = \frac{\boxed{1}}{\boxed{2}}$$

$$\frac{dy}{dx} = \frac{Differentiate\boxed{1}\times keep\boxed{2} - Differentiate\boxed{2}\times keep\boxed{1}}{\left(\boxed{2}\right)^2}$$

$$y = \frac{u}{v}, \boxed{\frac{dy}{dx} = \frac{\frac{du}{dx}v - \frac{dv}{dx}u}{v^2}}$$

e.g.1:

Differentiate $y = \frac{2x}{x+1}$ with respect to x.

Solution:

$y = \frac{2x}{x+1}$

$\frac{dy}{dx} = \frac{2(x+1)-1(2x)}{(x+1)^2}$

$\frac{dy}{dx} = \frac{2x+2-2x}{(x+1)^2}$

$\frac{dy}{dx} = \frac{2}{(x+1)^2}$ #

e.g.2:

Differentiate $y = \frac{x^2}{2x-1}$ with respect to x.

Solution:

$y = \frac{x^2}{2x-1}$

$\frac{dy}{dx} = \frac{2x(2x-1)-2(x^2)}{(2x-1)^2}$

$\frac{dy}{dx} = \frac{4x^2-2x-2x^2}{(2x-1)^2}$

$\frac{dy}{dx} = \frac{2x^2-2x}{(2x-1)^2}$

$\frac{dy}{dx} = \frac{2x(x-1)}{(2x-1)^2}$ #

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations [Approximation] *[hampir Sama Dengan]*

** The formulae can used in sections : *Approximate value, small changes, and percentages of small changes.*

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\boxed{\delta y \approx \frac{dy}{dx} \times \delta x} \rightarrow \textit{first formula}$$

Proof:

$y = x^2$ .... {1}

$y + \delta y = (x + \delta x)^2$

$y + \delta y = x^2 + 2x(\delta x) + (\delta x)^2$ ......{2}

{2} − {1},

$\delta y = 2x(\delta x) + (\delta x)^2$

$(\div \delta x)$

$\frac{\delta y}{\delta x} = 2x + (\delta x)$

But $\frac{dy}{dx} = 2x$

$\therefore \frac{\delta y}{\delta x} = \frac{dy}{dx} + \delta x$

$\begin{pmatrix} 1000 = 999 + 1 \\ 1000 \approx 999 \end{pmatrix}$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

therefore $\boxed{\delta y \approx \frac{dy}{dx} \times \delta x}$

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations [Approximation] *[hampir Sama Dengan]*

::::: v 9.031 Approximate Value

e.g.1:

Find the approximate value of $\sqrt{26}$ by using the calculus method.

Solution:

Let $y = \sqrt{x}$

$y = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

when $x = 25$, $\delta x = 1$ $\boxed{25+1=26}$

$y = \sqrt{25}$

$y = 5$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$\delta y \approx \frac{1}{2\sqrt{x}} \times 1$

$\delta y \approx \frac{1}{2\sqrt{25}} \times 1$

$\delta y \approx \frac{1}{2(5)} \times 1$

$\delta y \approx 0.1$

The approximate value for $\sqrt{26}$

$= y + \delta y$

$= 5 + 0.1$

$= 5.1$#

The value of $\sqrt{26}$ in the calculator is 5.09901954

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e.g.2:

Find the approximate value of 3.9^2 by using the calculus method.

Solution:

Let $y = x^2$

$\frac{dy}{dx} = 2x$

when $x = 4$, $\delta x = -0.1$ $\boxed{4-0.1=3.9}$

$y = 4^2$

$y = 16$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$\delta y \approx 2x \times -0.1$

$\delta y \approx 2(4) \times -0.1$

$\delta y \approx 8 \times -0.1$

$\delta y \approx -0.8$

The approximate value for 3.9^2

$= y + \delta y$

$= 16 + (-0.8)$

$= 15.2$ #

The value of 3.9^2 in the calculator is 15.21

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations [Approximation] *[hampir Sama Dengan]*

:::: v 9.031 Small Changes [approximated]

e.g.1:

Find the small changes in an area of circle when the radius increases from 2 cm to 2.1 cm by using the calculus method.

Solution:

Calculus Method:

$L = \pi r^2$

$\frac{dL}{dr} = 2\pi r$

when $r = 2, \delta r = 0.1$

$\delta L = \frac{dL}{dr} \times \delta r$

$\delta L = 2\pi r \times 0.1$

$\delta L = 2\pi(2) \times 0.1$

$\delta L = 0.4\pi$ #

Compare with the calculus method:

$L = \pi r^2$

when $r = 2$,	when $r = 2.1$,
$L = \pi(2)^2$	$L = \pi(2.1)^2$
$L = 4\pi \text{ cm}^2$	$L = 4.41\pi \text{ cm}^2$

Increases In L

$= 4.41\pi - 4\pi$

$= 0.41\pi$ #

e.g.2 (*Inverse Concepts*):

If $k = 2(9+h^2)$, find the small change of h when the k increases from 26 to 26.3 where $h > 0$.

Solution:

$k = 2(9+h^2)$

$k = 18+2h^2$

$\frac{dk}{dh} = 4h$

when $k = 26, \delta k = 0.3$

$\delta h \approx \frac{dh}{dk} \times \delta k$

$\delta h \approx \frac{1}{4h} \times 0.3$

$\delta h \approx \frac{1}{4(2)} \times 0.3$

$\delta h \approx 0.0375\#$

$k = 26$

$k = 18+2h^2$

$18+2h^2 = 26$

$2h^2 = 8$

$h^2 = 4$

$h = 2\ (h>0)$

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations [Approximation] [hampir Sama Dengan]

:::: v 9.033 Percentages Of Small Changes [approximated]

e.g.1:

Given that $k = 2(9+h^2)$, if x increases by $P\%$ when $x = 5$, find the percentages of small change for y.

Solution:

$y = 3x-2x^2$

$\frac{dy}{dx} = 3-4x$

when $x = 5, \delta x = \frac{P}{100} \times 5 = \frac{5P}{100}$

$y = 3(5)-2(5)^2$

$y = 15-50$

$y = -35$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$\delta y \approx (3-4x) \times \left(\frac{5P}{100}\right)$

$\delta y \approx (3-4(5)) \times \left(\frac{5P}{100}\right)$

$\delta y \approx (-17) \times \left(\frac{5P}{100}\right)$

$\delta y \approx -\frac{85P}{100}$

Percentages Of Small Change For y

$= \frac{\delta y}{y} \times 100\%$

$= \delta y \times \frac{1}{y} \times 100\%$

$= \left(-\frac{85P}{100}\right) \times \left(\frac{1}{(-35)}\right) \times 100\%$

$= \frac{17}{7} P\%\#$

e.g.2:

Given that $k = 12-4x+5x^2$, if x increases by $P\%$ when $x = 6$, find the percentages of small change for y.

Solution:

$y = 12-4x+5x^2$

$\frac{dy}{dx} = -4+10x$

when $x = 6, \delta x = \frac{P}{100} \times 6 = \frac{6P}{100}$

$y = 12-4(6)+5(6)^2$

$y = 168$

$\delta y \approx \frac{dy}{dx} \times \delta x$

$\delta y \approx (-4+10(6)) \times \left(\frac{6P}{100}\right)$

$\delta y \approx (56) \times \left(\frac{6P}{100}\right)$

$\delta y \approx \frac{336}{100} P$

Percentages Of Small Change For y

$= \frac{\delta y}{y} \times 100\%$

$= \delta y \times \frac{1}{y} \times 100\%$

$= \left(\frac{336P}{100}\right) \times \left(\frac{1}{168}\right) \times 100\%$

$= \frac{336}{168} P\%$

$= 2P\#$

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Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations [Approximation] *[hampir Sama Dengan]*

:::: v 9.034 Rates Of Change

(i) Rates Of Increases

$$x = \frac{dx}{dt}$$

(ii) The meaning Of Rates

$\frac{dx}{dt} = 3$ cm/s or 3 cm s^{-1} means that x is increases by 3 cm per seconds.

**(iii) Chain Rules: $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

e.g.

Given that $v = 3x + 2$ and $y = v^2 + 1$, find the rates of change of x when y is changing at the rate of 0.5 unit/s at $v = 3$.

Solution:

$v = 3x + 2$
$y = v^2 + 1$

Substitutes v,

$y = (3x+2)^2 + 1$

$y = (9x^2 + 12x + 4) + 1$

$y = 9x^2 + 12x + 5$

$\frac{dy}{dx} = 18x + 12$

$v = 3x + 2$

when $v = 3$

$3x + 2 = 3$

$3x = 1$

$x = \frac{1}{3}$

$\frac{dy}{dx} = 0.5$ unit/s

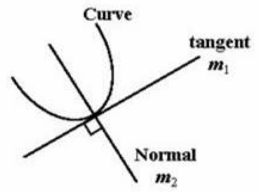
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$0.5 = \left(18\left(\frac{1}{3}\right) + 12\right) \times \frac{dx}{dt}$

$0.5 = 18 \times \frac{dx}{dt}$

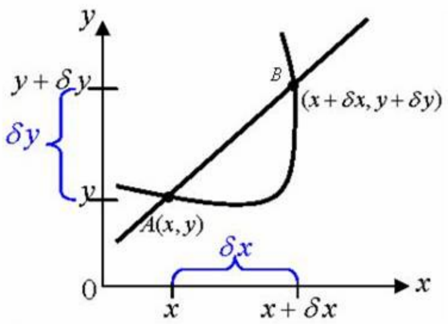
$\frac{dx}{dt} = \frac{1}{36}$ #

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations :::: v 9.035 Curves, Tangents, and Normal



$m_1 \times m_2 = -1$

$\frac{dy}{dx}$ = gradient of the tangent $\Leftrightarrow$ gradient of the curve $\Leftrightarrow$ gradient function



Proof:

$x_1 = x,\ x_2 = (x + \delta x),\ y_1 = y,\ x_2 = (y + \delta y)$

$m_{AB} = \frac{y_1 - y_2}{x_1 - x_2}$

$m_{AB} = \frac{y - (y + \delta y)}{x - (x + \delta x)}$

$m_{AB} = \frac{\delta y}{\delta x}$

$\frac{\delta y}{\delta x} = m_{AB}$

$\lim_{\delta x \to 0}\left(\frac{\delta y}{\delta x}\right)$ = gradient of tangent at point A.

$\left(\frac{\delta y}{\delta x}\right)$ = gradient of tangent at point B.

Topic 9 Differentiation :::::::::: v 9.03 Usages Of Differentiations

::::: v 9.036 Stationary Points, Turning Points, Maximum Points, Minimum Points And Point Of Inflexion

(i) Stationary Points

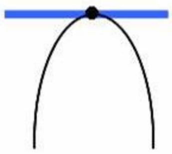
(a) for stationary points:

$\frac{dy}{dx} = 0$

(b) for stationary points: $\frac{dy}{dx} = 0$

(i) All Maximum Point

$\frac{dy}{dx} = 0$

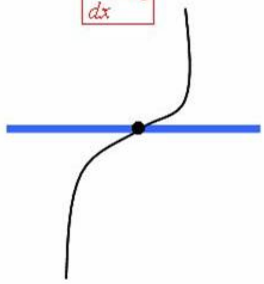


(ii) All Minimum Point

$\frac{dy}{dx} = 0$

(iii) Some Of the point of inflexion

$\frac{dy}{dx} = 0$

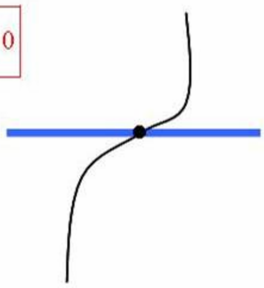


(ii) Points Of Inflexion [Point at the middle of "S"]

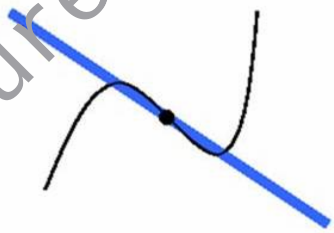
(1) Point at the middle of alphabet "S".

(2) Have 2 types of point of inflexion.

(i) $\frac{dy}{dx} = 0$



(ii) $\frac{dy}{dx} \neq 0$



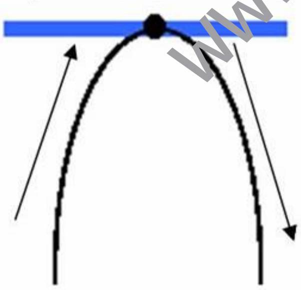
*(iii) Turning Point [Must changes the direction]

Point Of Inflexion not include In Turning Point

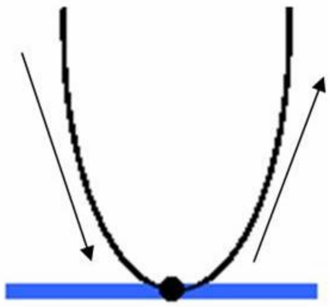
(1) $\frac{dy}{dx} = 0$

(2) 2 types of turning points.

Maximum Point



Minimum Point



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*(iv) The Condition Of Maximum Points

e.g.

Find the stationary point for the curve $y = 2x - x^2$ and determine the characteristics.

Solution:

$$\frac{dy}{dx} = 2 - 2x = 0$$

$$2x = 2$$

$$x = 1$$

when $x = 1, y = 2(1) - (1)^2 = 1$

stationary point $= (1,1)$

$$\frac{d^2y}{dx^2} = -2 < 0$$

$\therefore$ The stationary point $(1,1)$ is a maximum point.#

$$\frac{dy}{dx} = 0$$
$$\frac{d^2y}{dx^2} < 0$$

*(v) The Condition Of Minimum Points

e.g.

Find the stationary point for the curve $y = x^2 - 6x$ and determine the characteristics.

Solution:

$$\frac{dy}{dx} = 2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

when $x = 3, y = (3)^2 - 6(3) = -9$

stationary point $= (3,-9)$

$$\frac{d^2y}{dx^2} = 2 > 0$$

$\therefore$ The stationary point (3,-9) is a minimum point.#

$$\frac{dy}{dx} = 0$$
$$\frac{d^2y}{dx^2} > 0$$

*(vi) The Condition Of Point Of Inflexion

e.g.

Find the point of inflexion for $y = x^3 + 1$.

Solution:

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 6x = 0$$

$$x = 0$$

when $x = 0, y = (0)^3 + 1 = 1$

$(0,1)$

$$\frac{d^3y}{dx^3} = 6 \neq 0$$

$\therefore (0,1)$ is the point of inflexion.#

$$\frac{d^2y}{dx^2} = 0$$
$$\frac{d^3y}{dx^3} \neq 0$$

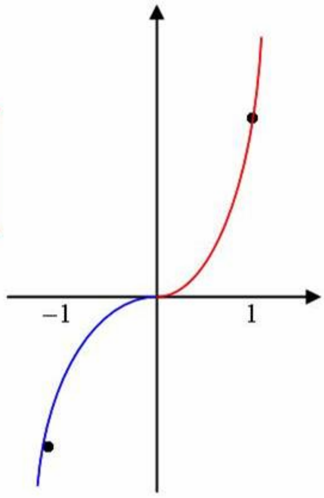
Note :

$$\frac{dy}{dx} = 3x^2$$

$$\frac{d^2y}{dx^2} = 6x$$

when $x = -1, \frac{d^2y}{dx^2} = 6(-1) = -6 < 0$

when $x = 1, \frac{d^2y}{dx^2} = 6(1) = 6 > 0$



*(*vi*) Maxima & Minima Problem

(a) For $V_{\max}$ or $V_{\min}$, $\frac{dV}{dx} = 0$

(b) If $V_{\max}$, then $\frac{d^2V}{dx^2} < 0$

(b) If $V_{\min}$, then $\frac{d^2V}{dx^2} > 0$

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