May/June 2002

3 A fair cubical die with faces numbered 1, 1, 1, 2, 3, 4 is thrown and the score noted. The area A of a square of side equal to the score is calculated, so, for example, when the score on the die is 3, the value of A is 9.

(i) Draw up a table to show the probability distribution of A. [3]

(ii) Find $E(A)$ and $Var(A)$. [4]

7 (i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

May/June 2003

2 A box contains 10 pens of which 3 are new. A random sample of two pens is taken.

(i) Show that the probability of getting exactly one new pen in the sample is $\frac{7}{15}$. [2]

(ii) Construct a probability distribution table for the number of new pens in the sample. [3]

(iii) Calculate the expected number of new pens in the sample. [1]

4 Kamal has 30 hens. The probability that any hen lays an egg on any day is 0.7. Hens do not lay more than one egg per day, and the days on which a hen lays an egg are independent.

(i) Calculate the probability that, on any particular day, Kamal's hens lay exactly 24 eggs. [2]

May/June 2004

3 Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X. [3]

x	1	2	3	4	5	6
$P(X = x)$						

(ii) Find $E(X)$. [2]

7 A shop sells old video tapes, of which 1 in 5 on average are known to be damaged.

(i) A random sample of 15 tapes is taken. Find the probability that at most 2 are damaged. [3]

(ii) Find the smallest value of n if there is a probability of at least 0.85 that a random sample of n tapes contains at least one damaged tape. [3]

May/June 2005

3 A fair dice has four faces. One face is coloured pink, one is coloured orange, one is coloured green and one is coloured black. Five such dice are thrown and the number that fall on a green face are counted. The random variable X is the number of dice that fall on a green face.

(i) Show that the probability of 4 dice landing on a green face is 0.0146, correct to 4 decimal places. [2]

(ii) Draw up a table for the probability distribution of X, giving your answers correct to 4 decimal places. [5]

May/June 2006

6 32 teams enter for a knockout competition, in which each match results in one team winning and the other team losing. After each match the winning team goes on to the next round, and the losing team takes no further part in the competition. Thus 16 teams play in the second round, 8 teams play in the third round, and so on, until 2 teams play in the final round.

(i) How many teams play in only 1 match? [1]

(ii) How many teams play in exactly 2 matches? [1]

(iii) Draw up a frequency table for the numbers of matches which the teams play. [3]

(iv) Calculate the mean and variance of the numbers of matches which the teams play. [4]

7 A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

(i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]

May/June 2007

7 A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

(i) Find the probability that the three peppers are all different colours. [3]

(ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]

(iii) The number of **green** peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. [5]

May/June 2008

6 Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X. [4]

x	0	1	2	3	4
$P(X = x)$		$\frac{1}{4}$			

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

7 A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

(i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]

(ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]

May/June 2009

1 The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

(i) Calculate the value of μ. [3]

(ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

2 Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that $P(X = 2) = \frac{5}{16}$. [2]

(ii) The table below shows the probability distribution of X.

x	2	3	4	5	6	7
$P(X = x)$	$\frac{5}{16}$	$\frac{1}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

Calculate $E(X)$ and $Var(X)$. [4]

Oct/Nov 2001

6 **65% of all watches sold by a shop have a digital display and 35% have an analog display.**

(i) Find the probability that, out of the next 12 customers who buy a watch, fewer than 10 choose one with a digital display. [4]

7 **A bag contains 7 orange balls and 3 blue balls. 4 balls are selected at random from the bag, without replacement. Let X denote the number of blue balls selected.**

(i) Show that $P(X = 0) = \frac{1}{6}$ and $P(X = 1) = \frac{1}{2}$. [4]

(ii) Construct a table to show the probability distribution of X. [3]

(iii) Find the mean and variance of X. [4]

Oct/Nov 2002

1 The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b. [1]

(ii) Given that $E(X) = 4$, find a and b. [3]

6 **(i)** A manufacturer of biscuits produces 3 times as many cream ones as chocolate ones. Biscuits are chosen randomly and packed into boxes of 10. Find the probability that a box contains equal numbers of cream biscuits and chocolate biscuits. [2]

(ii) A random sample of 8 boxes is taken. Find the probability that exactly 1 of them contains equal numbers of cream biscuits and chocolate biscuits. [2]

Oct/Nov 2003

4 Single cards, chosen at random, are given away with bars of chocolate. Each card shows a picture of one of 20 different football players. Richard needs just one picture to complete his collection. He buys 5 bars of chocolate and looks at all the pictures. Find the probability that

(i) Richard does not complete his collection, [2]

(ii) he has the required picture exactly once, [2]

(iii) he completes his collection with the third picture he looks at. [2]

8 A discrete random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	$3c$	$4c$	$5c$	$6c$

(i) Find the value of the constant c. [2]

(ii) Find $E(X)$ and $Var(X)$. [4]

(iii) Find $P(X > E(X))$. [2]

Oct/Nov 2004

6 A box contains five balls numbered 1, 2, 3, 4, 5. Three balls are drawn randomly at the same time from the box.

(i) By listing all possible outcomes (123, 124, etc.), find the probability that the sum of the three numbers drawn is an odd number. [2]

The random variable L denotes the largest of the three numbers drawn.

(ii) Find the probability that L is 4. [1]

(iii) Draw up a table to show the probability distribution of L. [3]

(iv) Calculate the expectation and variance of L. [3]

7 **(i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

Oct/Nov 2005

5 A box contains 300 discs of different colours. There are 100 pink discs, 100 blue discs and 100 orange discs. The discs of each colour are numbered from 0 to 99. Five discs are selected at random, one at a time, with replacement. Find

(i) the probability that no orange discs are selected, [1]

(ii) the probability that exactly 2 discs with numbers ending in a 6 are selected, [3]

(iii) the probability that exactly 2 orange discs with numbers ending in a 6 are selected, [2]

(iv) the mean and variance of the number of pink discs selected. [2]

6 In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

(iii) Draw up a probability distribution table for Mario's profit. [3]

(iv) Calculate his expected profit. [2]

Oct/Nov 2006

2 The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.26	q	$3q$	0.05	0.09

(i) Find the value of q. [2]

(ii) Find $E(X)$ and $Var(X)$. [3]

7 A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

(i) equal numbers of large and small bands, [2]

(ii) more than 17 small bands. [3]

Oct/Nov 2007

2 The random variable X takes the values -2, 0 and 4 only. It is given that $P(X = -2) = 2p$, $P(X = 0) = p$ and $P(X = 4) = 3p$.

(i) Find p. [2]

(ii) Find $E(X)$ and $Var(X)$. [4]

7 Box A contains 5 red paper clips and 1 white paper clip. Box B contains 7 red paper clips and 2 white paper clips. One paper clip is taken at random from box A and transferred to box B. One paper clip is then taken at random from box B.

(i) Find the probability of taking both a white paper clip from box A and a red paper clip from box B. [2]

(ii) Find the probability that the paper clip taken from box B is red. [2]

(iii) Find the probability that the paper clip taken from box A was red, given that the paper clip taken from box B is red. [2]

(iv) The random variable X denotes the number of times that a red paper clip is taken. Draw up a table to show the probability distribution of X. [4]

Oct/Nov 2008

7 A fair die has one face numbered 1, one face numbered 3, two faces numbered 5 and two faces numbered 6.

(i) Find the probability of obtaining at least 7 odd numbers in 8 throws of the die. [4]

The die is thrown twice. Let X be the sum of the two scores. The following table shows the possible values of X.

		Second throw					
		1	3	5	5	6	6
	1	2	4	6	6	7	7
	3	4	6	8	8	9	9
First throw	5	6	8	10	10	11	11
	5	6	8	10	10	11	11
	6	7	9	11	11	12	12
	6	7	9	11	11	12	12

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Calculate $E(X)$. [2]

(iv) Find the probability that X is greater than $E(X)$. [2]

Oct/Nov 2009/11

1 The mean number of defective batteries in packs of 20 is 1.6. Use a binomial distribution to calculate the probability that a randomly chosen pack of 20 will have more than 2 defective batteries. [5]

2 The probability distribution of the random variable X is shown in the following table.

x	−2	−1	0	1	2	3
$P(X = x)$	0.08	p	0.12	0.16	q	0.22

The mean of X is 1.05.

(i) Write down two equations involving p and q and hence find the values of p and q. [4]

(ii) Find the variance of X. [2]

Oct/Nov 2009/12

2 Two unbiased tetrahedral dice each have four faces numbered 1, 2, 3 and 4. The two dice are thrown together and the sum of the numbers on the faces on which they land is noted. Find the expected number of occasions on which this sum is 7 or more when the dice are thrown together 200 times. [4]

5 In a particular discrete probability distribution the random variable X takes the value $\frac{120}{r}$ with probability $\frac{r}{45}$, where r takes all integer values from 1 to 9 inclusive.

(i) Show that $P(X = 40) = \frac{1}{15}$. [2]

(ii) Construct the probability distribution table for X. [3]

(iii) Which is the modal value of X? [1]

(iv) Find the probability that X lies between 18 and 100. [2]

May/June 2010/61

1 The probability distribution of the discrete random variable X is shown in the table below.

x	-3	-1	0	4
$P(X = x)$	a	b	0.15	0.4

Given that $E(X) = 0.75$, find the values of a and b. [4]

5 In the holidays Martin spends 25% of the day playing computer games. Martin's friend phones him once a day at a randomly chosen time.

(i) Find the probability that, in one holiday period of 8 days, there are exactly 2 days on which Martin is playing computer games when his friend phones. [2]

(ii) Another holiday period lasts for 12 days. State with a reason whether it is appropriate to use a normal approximation to find the probability that there are fewer than 7 days on which Martin is playing computer games when his friend phones. [1]

(iii) Find the probability that there are at least 13 days of a 40-day holiday period on which Martin is playing computer games when his friend phones. [5]

May/June 2010/62

6 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

(ii) Show that $E(X) = \frac{8}{7}$ and calculate $Var(X)$. [3]

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

May/June 2010/63

5 Set A consists of the ten digits 0, 0, 0, 0, 0, 0, 2, 2, 2, 4.

Set B consists of the seven digits 0, 0, 0, 0, 2, 2, 2.

One digit is chosen at random from each set. The random variable X is defined as the sum of these two digits.

(i) Show that $P(X = 2) = \frac{3}{7}$. [2]

(ii) Tabulate the probability distribution of X. [2]

(iii) Find $E(X)$ and $Var(X)$. [3]

(iv) Given that $X = 2$, find the probability that the digit chosen from set A was 2. [2]

Oct/Nov 2010/61

7 Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X.

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $\text{Var}(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

Oct/Nov 2010/62

1 The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

x	1	4	5	7	9
$P(X = x)$	$4p$	$5p^2$	$1.5p$	$2.5p$	$1.5p$

Find p. [3]

6 **(i)** State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

(ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]

(iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

Oct/Nov 2010/63

2 In a probability distribution the random variable X takes the value x with probability kx, where x takes values 1, 2, 3, 4, 5 only.

(i) Draw up a probability distribution table for X, in terms of k, and find the value of k. [3]

(ii) Find $E(X)$. [2]